The Open University of Sri Lanka

Foundation Course in Science – Level 02

~~Open Book Test~~ 2013/2014

PAE/PAF 2201- Combined Mathematics I

Duration: - One and half Hours.

Date:-11.12.2013 **Time:-1.30p.m-3.00p.m**

1) (i)

(a) Prove that

$$\frac{Sin(A-B)}{CosACosB}+\frac{Sin(B-C)}{CosBCosC}+\frac{Sin(C-A)}{CosCCosA}=0.$$

(b) If $Cosy = Sin(x+y)$,then prove that

$$\tan y = \sec x - \tan x.$$

(ii) Find the general solution of the equation

$$\tan x + \tan 2x = \sqrt{3}(1-\tan x \tan 2x).$$

(iii)Let $f(x) = 2\text{Sin}^2\left(\frac{x}{2}\right)+2\sqrt{3}Sin\left(\frac{x}{2}\right)Cos\left(\frac{x}{2}\right)+4Cos^2\left(\frac{x}{2}\right)$. Express $f(x)$ in the form $aSin(x+\theta)+b$ where $a(>0)$ and $\theta(0<\theta<\pi/2)$ are constant to be determined, Hence find the general solution of the equation $f(x)=5$.

2) (i) Let $f(x) = px^3+qx^2-11x+6$, where $p,q \in \mathbb{R}$ if $(x-1)$ is a factor of $f(x)$, and the remainder when $f(x)$ is divided by $(x-4)$ is -6, find the values of p and q, Also find the other two linear factors.

(ii) Let α and β be the roots of the equation $x^2+bx+c=0$ and γ and δ be the roots of the equation $x^2+mx+n=0$ where $b,c,m,n \in \mathbb{R}$.

(a) Find $(\alpha-\beta)^2$ in terms of b and c, hence write down $(\gamma-\delta)^2$ in terms of m and n.

Deduce that if $\alpha+\gamma=\beta+\delta$, then $b^2-4c=m^2-4n$.

3) (i) Let $U_r = \dfrac{3(6r+1)}{(3r-1)^2(3r+2)^2}$ for $r \in \mathbb{Z}^+$ and let $S_n = \sum_{r=1}^{n} U_r$ for $n \in \mathbb{Z}^+$. Find the values of the constant A and B such that

$$U_r = \frac{A}{(3r-1)^2} + \frac{B}{(3r+2)^2} \text{ for } r \in \mathbb{Z}^+.$$

Hence show that $S_n = \dfrac{1}{4} - \dfrac{1}{(3n+2)^2}$ for $n \in \mathbb{Z}^+$.

Is the infinite series $\sum_{r=1}^{\infty} U_r$ convergent?

Find the value of $\sum_{r=1}^{\infty} U_r$.

(ii) (a) The complex numbers $Z_1 = \dfrac{\sqrt{3}}{2} + i\dfrac{1}{2}$ and $Z_2 = \dfrac{-1}{2} + i\dfrac{\sqrt{3}}{2}$ are represented in an Argand diagram by points A and B respectively. Find $Arg(Z_1)$ and $Arg(Z_2)$. Given that $OACB$ is a square in Argand diagram, where O is the origin, find the modulus and the argument of the complex number represented by C.

(b) Find the least value of $|Z|$ subjected to the condition $\arg(z+1) = \pi/6$.