The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination 2016/2017
Level 04 Applied Mathematics
APU 2140/APE 5140– Statistical Distribution Theory

Duration: - Two hours

Date: - 29-07-2017 **Time: - 9.30 a.m to 11.30 a.m**

Non programmable calculators are permitted. Statistical tables are provided.
Answer four questions only.

1.

The density function of a particular random variable X is given by

$$f(x)\begin{cases} \dfrac{x}{2k} & ;\ when\ 0 < x \leq 5 \\ \dfrac{(10-x)}{2k} & ;\ when\ 5 < x < 10 \\ 0 & ;\quad otherwise \end{cases}$$

(i) Find the value of $\boldsymbol{k}$.
(ii) Find the mean of $\boldsymbol{X}$.
(iii) Find cumulative distribution function of X.
(iv) Find the median of X.
(v) Find Pr(2 < **X** < 7)

2.

(a) Some traffic lights have three phases: ***stop 35%*** of the time, ***wait 10% of the time*** and ***go 55%*** of the time. Assume that a person only crosses a traffic light when it is in the ***go*** position. Nayomi has to pass 4 such traffic lights on her way to school.

(i) Find the probability that Nayomi has to ***wait or stop*** at 2 traffic lights on her way to school.

(ii) Find the probability that Nayomi has to ***wait or stop*** for the first time at the third traffic light she meets on her way to school.

(iii) Find the probability that Nayomi has to cross the third and fourth traffic lights on her way to school.

(b) The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 9.

(i) Find the probability of observing four or more accidents on this stretch of road during next month.

(iii) Assuming that a month has four weeks find the probability of observing two to four accidents on this stretch of road during the first 14 days of next month.

3.

(a) Random variable ***X*** has the probability mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0,1,2,3, \dots\dots\dots$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i), show that $E(X) = \lambda$ and $Var(X) = \lambda$

(b) Suppose that X_1, X_2, X_3, X_4 are independent random variables described as

$$X_1 \sim N(3,4) \qquad X_2 \sim N(5,9) \qquad X_3 \sim exp(3) \qquad X_4 \sim gamma(3,3)$$

Find the following probabilities. Show your calculations and state the justifications clearly. You may use the gamma table at the end of the paper wherever necessary.

(i) $\Pr[(2X_1 + 3X_2) < 25\ )]$

(ii) $\Pr[(X_3 + X_4) > 3\ )]$

4.

Life time of a light bulb manufactured by ABC Company is normally distributed with mean 600 days and a standard deviation of 100 days.

(i) Find the probability that life time of a randomly selected light exceeds two years.

(ii) The production manager of the ABC Company plans to set a warranty period (in days) such that 95% of the bulbs should not fail during the warranty period. Calculate the warranty period.

(iii) Random sample of 10 bulbs from the above population were tested and sample mean $\bar{X}$ was estimated. Find the probability that sample mean $\bar{X}$ exceeds 550 days.

(iv) From past experience it is known that mean salary of employees in ABC company is Rs. 50000 with standard deviation of Rs. 2000. Suppose a sample of 100 employees were selected for a survey. Find the probability that mean salary of the selected sample of employees will be less than Rs.49800? Clearly state any assumptions or theorems used in the answering to this part.

5.

(a) A manufacturer of electronic calculators offers a one year warranty. If the calculator fails for any reason during this period, it is replaced. The time to failure is well modeled by the following probability distribution.

$$f(x) = 0.125e^{-0.125x} \quad ; x > 0$$

What percentages of the calculators will fail within the warranty period?

(b) Suppose that a sample of n = 1,600 tires of the same type are obtained at random from an ongoing production process in which 8% of all such tires produced are defective. What is the probability that in such a sample 150 or fewer tires will be defective?

(c) A quality characteristic of a product is normally distributed with mean 7 and standard deviation one. Specifications of this characteristic are $6 \leq x \leq 8$. A unit that falls in this specification on this quality characteristic results in a profit C_0.

However if X < 6 the profit is $-C_1$, wherever if X > 8 the profit is $-C_2$. Find the expected profit.

6.

Measurements for the length and width of a rectangular plastic covers for CDs are rounded to the nearest millimeter. Let ***X*** denotes the length (mm) and ***Y*** denotes the width (mm). The joint distribution of ***X*** and ***Y*** is given below.

P(X=x, Y=y)		***X***		
		129	**130**	**131**
Y	**15**	0.12	0.42	0.06
	16	0.06	0.28	*k*

(i) Find the value of *k*.

(ii) Calculate the following probabilities.

(I) $Pr(X<131, Y=16)$ (II) $Pr(Y=16)$ (iii) $Pr(X<130)$

(iii) Find the marginal distribution of *X*.

(iv) Kamal says that "Length and width of a rectangular plastic covers for CDs are independent ". State whether the Kamal's statement is true or false. Justify your answer.

(v) Calculate $Pr\ (X \geq 130 | Y = 16)$.

(vi) Calculate $E\ (X | Y = 16)$.

Left tail values of Standard Gamma Table

W - gamma(α,1)

This table contain the probabilities Pr(W ≤ w)

w	α					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432