THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
FINAL EXAMINATION 2015/2016
APPLIED MATHEMATICS-LEVEL 05
APU3146/APE5146 - Operations Research
DURATION: TWO HOURS

Date: 20.01.2017 **Time: 09.30 a.m- 11.30 a.m**

ANSWER FOUR QUESTIONS ONLY.

01. (a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point.

Player B

Player A		B_1	B_2
	A_1	a	$-b$
	A_2	$-c$	d

where a, b, c, d are all non-negative.

Prove that the optimal strategies are:

$$A = \begin{bmatrix} A_1 & A_2 \\ \frac{c+d}{a+b+c+d} & \frac{a+b}{a+b+c+d} \end{bmatrix} \qquad B = \begin{bmatrix} B_1 & B_2 \\ \frac{b+d}{a+b+c+d} & \frac{a+c}{a+b+c+d} \end{bmatrix} \text{ and}$$

Value of the game is $v = \dfrac{ad - bc}{a+b+c+d}$.

(b) The manager of a multinational company and the Union of workers are preparing to sit down at the bargaining table to work out the details of a new contract for the workers. Each side has developed certain proposals for the contents of the new contract. Union proposals are called "Proposal *I* ", "Proposal *II* " and "Proposal *III* ". Manager's proposals are called "Contract *A*", "Contract *B*" and "Contract *C*". Both parties are aware of the financial aspects of each proposal-contract combination. The reward matrix is:

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Proposal	**Contract**		
	A	*B*	*C*
I	8.5	7.0	7.5
II	12.0	9.5	10.0
III	9.0	11.0	8.0

(i) Is there a saddle point? Justify your answer.

(ii) Find the strategies which are dominated by other strategies, and reduce the size of the reward matrix.

(iii) Formulate a Linear Programming model to determine the optimum strategy of the Union and the optimum strategy of the Manager.

02. (a) Briefly explain the following terms:

(i) Queue discipline

(ii) Service mechanism

(iii) Service channel

(b) Patients arrive at the Government hospital for emergency service at the rate of one every hour. Currently, only one emergency case can be handled at a time. Patients spend on average of 20 minutes receiving emergency care.

(i) What is the probability that a patient arriving at the hospital will have to wait?

(ii) Find the average length of the queue that forms.

(iii) Find the average time a patient spends in the system.

(iv) What is the probability that there will be five or more patients waiting for the service?

(v) Determine the fraction of the time that there are no patients.

(vi) Find the average service time need to be decreased to keep the average time in the system less than 25 minutes.

03. There are two clerks in a University to receive dues from the students. One clerk handles with 1st and 2nd year students and the other clerk handles with 3rd and 4th year students. It has been found that the service time distributions for students are exponential with mean service time 5 minutes per student. First and 2nd year students are found to arrive in a Poisson fashion throughout the day with mean arrival rate 8 per hour. Third and 4th year students also arrive in a Poisson fashion with mean arrival rate 6 per hour.

(a) What would be the effect on the average waiting time for students if each clerk could handle any student who comes from any year?

(b) What would be the effect if this could only be accomplished by increasing the service time 6 minutes?

04. A car servicing station has two bags where service can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with mean 6 minutes. Find

(a) The average number of cars in the service station,

(b) The average number of cars in the system during the peak hours,

(c) The average waiting time of a car spends in the system,

(d) The average number of cars per hour that cannot enter the station because of full capacity.

05. (a) Define the term inventory.

(b) What are the advantages and disadvantages of having inventories?

(c) Formulate the Economic Order Quantity (EOQ) model in which demand is not uniform and production rate is infinite.

Let $t_1, t_2, \ldots, t_n$ denote the times of successive production runs such that

$$t_1 + t_2 + \ldots\ldots\ldots + t_n = 1 \text{ year}$$

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(d) A company uses annually 24000 units of a raw material which costs Rs. 1.25 per unit. Placing each order costs Rs. 22.50, and the carrying cost is 5.4 per cent per year of the average inventory. Find the Economic Order Quantity and the total inventory cost.

06. (a) Briefly explain the following terms in inventory management:

(i) Carrying cost

(ii) Shortage cost

(iii) Ordering cost

(b) Derive EOQ model for deterministic demand when replenishment rate is infinite and shortages are permitted.

(c) A particular item has a demand of 9000 units per year. The cost of one procurement is Rs.100 and the holding cost per unit is Rs. 2.40per year. The replacement is instantaneous and the cost of shortage is Rs. 5 per unit per year. Determine

(i) the lot size,

(ii) the number of orders per year,

(iii) the time between orders and

(iv) the total cost per year if the cost of one unit is Rs.1.

Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq \text{n}) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad E(v) = \frac{1}{\mu-\lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

(M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho=1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1-N\rho^{N-1}+(N-1)\rho^N\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(n) = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)\left(1-\rho^{N+1}\right)} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ;1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ;n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1} \qquad E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;0 \le n \le C \\ \dfrac{1}{C^{n-1}C!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;C < n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{C-1}\dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n + \dfrac{1}{C!}\left(\dfrac{\lambda}{\mu}\right)^C \left\{1-\left(\dfrac{\lambda}{C\mu}\right)^{N-C+1}\right\}\dfrac{C\mu}{C\mu-1}\right]^{-1} & ;\dfrac{\lambda}{C\mu} \ne 1 \\ \left[\displaystyle\sum_{n=0}^{C-1}\dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n + \dfrac{1}{C!}\left(\dfrac{\lambda}{\mu}\right)^C (N-C+1)\right]^{-1} & ;\dfrac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \dbinom{K}{n}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;\ 0 \le n < R \\ \dbinom{K}{n}\dfrac{n!}{R^{n-R}R!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;\ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda[K-E(n)]}$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda[K-E(n)]}$$

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