The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2014/2015

APU 2144/APE4144- Applied Linear Algebra and Differential Equations

APPLIED MATHEMATICS-LEVEL 04



Duration: Two Hours.

Date: 04.11.2015

Time: 09.30 a.m. - 11.30 a.m.

Answer FOUR questions only.

1. (a) Find a value of k so that the matrix A where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 3 & k \end{bmatrix}$$

is singular.

(b) Find the rank of the matrix
$$B$$
 where $B = \begin{bmatrix} 2 & -2 & 4 & 3 \\ 1 & -1 & 2 & 2 \\ 2 & -2 & 1 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$.

(c) Find the values of a and b for which the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b.$$

has (i) no solution

- (ii) a unique solution
- (iii) an infinitely many solutions.



- 2. (a) Let A B and C be three matrices. Prove each of the following:
 - (i) If B and C are two inverses of A, then B = C.

(ii)
$$(AB)^{-1} = B^{-1}A^{-1}$$
.

(iii)
$$\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$$

(iv)
$$(A^{-1})^{-1} = A$$
.

(b) Find the orthogonal transformation which transforms the quadratic form

$$2(x_1x_2 + x_2x_3 + x_3x_1)$$

to canonical form (or "sum of squares form").

3. Find the general solutions of each of the systems of simultaneous differential equations, given below. Here, the dots signify a standard notation involving the parameter t.

(a)
$$\dot{x}_1 = x_1 - x_2 - x_3$$

$$\dot{x}_2 = x_1 + 3x_2 + x_3$$

$$\dot{x}_3 = -3x_1 + x_2 - x_3$$

(b)
$$\dot{x}_1 = -2x_1 + 10x_2 + 48$$

$$\dot{x}_2 = 10x_1 - 2x_2 + 91e^t$$

(c)
$$\ddot{x} = 8x - 5y$$

$$\ddot{y} = 10x - 7y$$

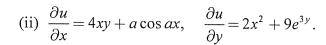
4. (a) Find a sinusoidal particular solution for the following system of partial differential equations:

$$\ddot{x}_1 + 2\ddot{x}_2 + \dot{x}_1 + x_1 - 3x_2 = \sin t$$

$$3\ddot{x}_1 + \ddot{x}_2 + 2\dot{x}_2 + 2x_1 + x_2 = \cos t - 2\sin t.$$

(b) Find the general solution of each of the following simultaneous partial differential equations:

(i)
$$\frac{\partial u}{\partial x} = 6x^2$$
, $\frac{\partial u}{\partial y} = 12y^2$.



(c) Find the general solution of the following differential equation:

$$x^{2} \frac{d^{2} y}{dx^{2}} - 5x \frac{dy}{dx} + 8y = 2x^{3}.$$

- 5. Consider the partial differential equation $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} x^2 u = 1 \quad (x \neq 0, y \neq 0).$
 - (a) With the transformation $\zeta = x$ and $\phi = \frac{1}{y} \frac{1}{x}$ find using the chain rule, expressions for $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ in terms of $\frac{\partial u}{\partial \zeta}$ and $\frac{\partial u}{\partial \phi}$.
 - (b) Hence, simplify the partial differential equation to the form $\frac{\partial u}{\partial \zeta} u = \frac{1}{\zeta^2}$.
 - (c) Solve this equation for $u(\zeta, \phi)$, and hence find the general solution of the original equation in terms of x and y.
- 6. Show that the general solution of the equation $\frac{\partial^2 u}{\partial x^2} 4x^2 \frac{\partial^2 u}{\partial y^2} \frac{1}{x} \frac{\partial u}{\partial x} = 0, \quad (x \neq 0)$

may be found by reducing it to the standard form $\frac{\partial^2 u}{\partial \zeta \partial \phi} = 0$, where $\zeta = x^2 - y$ and

 $\phi = x^2 + y$. Use the method of characteristics to derive expressions for ζ and ϕ .