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The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2013/2014
Applied Mathematics
AMU2185/ AME4185 – Numarical Methods
Duration :- Two Hours

Date :-20.11.2014 **Time:-01.30 p.m.-03.30 p.m.**

Answer Four Questions Only.

1. (a) Briefly explain simple iterative method.

   (b) Let $x=\xi$ be a root of $f(x)=0$ such that $\xi \in I$ where I is an interval. Let $\phi(x)-x \equiv f(x)$ such that $\phi(x)$ and $\phi'(x)$ be continuous in I. Prove that if $|\phi'(x)|<1$ for all $x \in I$ the sequence $x_1, x_2, \ldots x_n$ defined by $\phi(x_n)=x_n$ converges to the root ξ provided that initial approximation $x_0 \in I$.

   (c) Using simple iterative method, find the root of the equation $\sin x - 3x + 1 = 0$ lying in the interval $[0, 0.5]$ and correct to four decimal places.

2. (a) Derive Newton- Raphson formula for solving the equation $f(x)=0$.

   (b) Show that Newton- Raphson method has quadratic convergence.

   (c) Using the Newton- Raphson method, find the root lies $[0.1, 0.2]$ of $x(1-\ln x)=0.5$ correct up to four decimal places.

   (d) Derive general formula to find $\sqrt{N}$ by Newton -Raphson method where N is a positive real number. Hence find $\sqrt{12}$.

3. (a) Prove that

   (i) $\Delta = E - 1$, (ii) $\left[\left(\frac{\Delta^2}{E}\right)e^x\right]\left[\frac{Ee^x}{\Delta^2 e^x}\right] = e^x$, where Δ, ∇, δ and E are the forward difference, the backward difference, the central difference and the shift operators respectively.

(b) Derive the Gregory- Newton forward interpolation formula.

(c) Hence, interpolate $f(22)$ corresponding to the data points (20, 12), (25, 15), (30, 20), (35, 27), (40, 39) and (45, 52).

4. (a) Prove that

(i) $E=(1-\nabla)^{-1}$, (ii) $\nabla=1-E^{-1}$,

where Δ, ∇ and E are the forward difference, the backward difference and the shift operators respectively.

(b) Derive the Gregory- Newton backward interpolation formula.

Hence, interpolate $f(42)$ corresponding to the data points (20, 354), (25, 332), (30, 291), (35, 260), (40, 231) and (45, 204).

5. (a) Derive the Lagrange's interpolation formula.

(b) Find the Lagrange polynomial (f) passing through the points (3, 168), (7, 120), (9, 72) (10, 63) and determine $f(6)$.

6. (a) If the polynomial $f(x) = x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n$ is divided by $x - \alpha$ the quotient is given by $q(x) = x^{n-1} + b_1x^{n-2} + \cdots + b_{n-2}x + b_{n-1}$, show that the remainder b_n is given by $b_n = \alpha b_{n-1} + a_n$.

(b) Explain Honer's scheme to determine the coefficients of the quotient $q(x)$ when a polynomial $f(x)$ is divided by $x - \alpha$.

(c) Use Honer's scheme to determine the coefficients of the quotient if

$f(x) = x^5 - 3x^4 + 4x^3 + 2x^2 - 10x - 4$ is divided by $x - 2$.

Hence show that $x - 2$ is a factor of $f(x)$.