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The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2013/2014

APU 2144/APE4144- Applied Linear Algebra and Differential Equations

APPLIED MATHEMATICS-LEVEL 04

Duration: Two Hours.

Date: 30.11.2014	**Time: 09.30 a.m. - 11.30 a.m.**

Answer FOUR questions only.

1. (a) Explain the consistency or the inconsistency of a system of m linear equations in n unknowns, making special mention of the ranks of certain related matrices.

   (b) Determine the non-singular matrices P and Q such that PAQ is the normal form for the matrix A.

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

   Hence find the rank of A.

   (c) For which rational numbers a and b does the following system have

   (i) No solution?

   (ii) A unique solution?

   (iii) Infinitely many solutions?

$$\begin{aligned} x-2y+3z&=4 \\ 2x-3y+az&=5 \\ 3x-4y+5z&=b \end{aligned}$$

   Justify your answers.

2. (a) If A and B are square matrices of the same order and A is symmetric, prove that $B^T AB$ is also symmetric.

(b) Prove that the inverse of a non–singular symmetric matrix A is symmetric.

(c) Transform the following quadratic form to a canonical form by an orthogonal transformation and state the corresponding model matrix.

$$5x_1^2 + 26x_2^2 + 10x_3^2 + 4x_2x_3 + 6x_1x_2 + 14x_1x_3.$$

3. (a) Find the general solution of each of the systems of simultaneous differential equations, given below. Here, the dots signify a standard notation.

(i)
$$\dot{x}_1 = 4x_1 + 3x_2 + x_3$$
$$\dot{x}_2 = -4x_1 - 4x_2 - 2x_3$$
$$\dot{x}_3 = 8x_1 + 12x_2 + 6x_3$$

(ii)
$$\ddot{x} = x - 4y$$
$$\ddot{y} = -x + y$$

(b) Find a sinusoidal particular solution for the following system of partial differential equations.

$$\ddot{x}_1 + 4x_1 + 2x_2 = 6\cos 2t$$
$$\ddot{x}_2 + x_1 + 9x_2 = 2\sin 2t.$$

4. (a) Find the general solution of each of the following simultaneous partial differential equations:

(i) $\dfrac{\partial u}{\partial x} = 3x^2, \quad \dfrac{\partial u}{\partial y} = 8y.$

(ii) $\dfrac{\partial u}{\partial x} = 3x^2 y - a\sin ax, \quad \dfrac{\partial u}{\partial y} = x^3 - e^{-y}.$

(b) Consider the first order partial differential equation $\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} + u = 2.$

(i) By defining new variables $\zeta,\ \phi$ such that $x = \zeta$ and $y = \phi - \zeta$, use the chain rule to find $\dfrac{\partial u}{\partial \zeta}$ in terms of $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$.

(ii) Hence transform the above partial differential equation to $\dfrac{\partial u}{\partial \zeta}+u=2$.

(iii) Solve this equation for $u(\zeta, \phi)$,and hence show that the general solution to the original equation is $u=2+e^{-x}f(x+y)$ where f is an arbitrary function.

5. (a) Find the equations of the characteristic curves for the partial differential equation

$$-2xy\frac{\partial u}{\partial x}+4x\frac{\partial u}{\partial y}+yu=4xy, \qquad x>0,\ y>0$$

and define new variables that could be used to simplify the equation.

(b) Hence obtain the general solution $u(x, y)$ of the partial differential equation given in part (a).

(c) Calculate $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ for the function $u(x, y)$ of part (b).

(d) Show that $\dfrac{\partial u}{\partial x}$, $\dfrac{\partial u}{\partial y}$ of part (c) and $u(x,y)$ of part (a) satisfies the partial differential equation in part (a).

(6) Show that the general solution of the equation $\dfrac{\partial^2 u}{\partial x^2}-4x^2\dfrac{\partial^2 u}{\partial y^2}-\dfrac{1}{x}\dfrac{\partial u}{\partial x}=0, \quad (x\neq 0)$

may be found by reducing it to the standard form $\dfrac{\partial^2 u}{\partial \zeta \partial \phi}=0$, where $\zeta=x^2-y$ and $\phi=x^2+y$. Use the method of characteristics to derive expressions for ζ and ϕ.