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The Open University of Sri Lanka
B.Sc Degree Programme
Level 05 - Final Examination 2008/2009
Pure Mathematics
PMU 3292/PME 5292 – Group Theory and Transformation - Paper II

Duration :- 2 1/2 Hours.

Date :- 05–01–2009. **Time:- 1.00 p.m. – 3.30 p.m.**

Answer FOUR questions only.

01.(a) Let H, K be subgroups of a group G of orders 3, 5 respectively. Prove that $H \cap K = \{1\}$

(b) Let $H_3 = 3\mathbb{Z}$ be the subgroup consisting of multiples of 3 in the group $(\mathbb{Z}, +)$. Find the right cosets of H_3 in $\mathbb{Z}$.

02. A subgroup K of a group G is said to be normal if for each $a \in G$, $Ka = aK$.

(i) Show that $A_3 = \{I, (1\ 2\ 3), (1\ 3\ 2)\}$ is a normal subgroup of S_3, where S_3 is the symmetric group of order 6.

(ii) Show that even though $A_3(1\ 2) = (1\ 2)A_3$, it is not true that $\rho(1\ 2) = (1\ 2)\rho$ for each ρ in A_3.

03. (a) For $a, b \in R$, $a \neq 0$, define $f : \mathbb{R} \to \mathbb{R}$ by $f_{ab}(x) = ax + b$. Let $G = \{f_{ab} \mid a, b \in \mathbb{R},\ a \neq 0\}$ and $N = \{f_{1b} \in G\}$. Prove that N is a normal subgroup of G.

(b) Prove that the subgroup N of G is a normal subgroup of G if and only if $gN = Ng$, $\forall g \in G$.

04. Determine which of the following mapping $f : ₵^* \mapsto \mathbb{R}^*$ are homomorphisms and which or not.

(i) $f(z) = |z|$ (ii) $f(z) = 2|z|$ (iii) $f(z) = \frac{1}{|z|}$

(iv) $f(z) = 1 + |z|$ (v) $f(z) = |z|^2$ (vi) $f(z) = 1$.

05. (a) $f: G \mapsto G$ is defined by

(i) $f(x) = x^2$ (ii) $f(x) = x^{-1}$

Decide which of the above mappings are homomorphisms. Justify your answer.

(b) Let $W = \left(\begin{bmatrix} x & -x \\ 0 & 0 \end{bmatrix} : x \in Q^* \right)$ be a group under matrix multiplication. Prove that $(W, \times) \cong (\mathbb{Q}^*, \times)$.

06. (a) Let G be a group such that $G/Z(G)$ is cyclic. Show that G is abelian, where $Z(G)$ is the centre of G.

(b) If $\mathbb{R}^*$ is the group of non zero real numbers under multiplication, then prove that $(\mathbb{R}^*, \cdot)$ is not isomorphic to $(\mathbb{R}, +)$.