THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. DEGREE PROGRAMME
FINAL EXAMINATION 2017/2018
APPLIED MATHEMATICS-LEVEL 05
APU3146/ADU5304 – OPERATIONAL RESEARCH
DURATION: TWO HOURS

**Date: 10.04.2019** **Time: 09.30 a.m- 11.30 a.m**

**ANSWER FOUR QUESTIONS ONLY.**

**Question 01**

(a) Define a game in game theory, explaining its properties.

(b) Define the two-person zero sum game. Write down the assumptions made in two-person zero sum games.

(c) There are two competing shoe shops *BATA* and *DSI* in a city. Both shops have equal reputation and the total number of customers is equally divided between the two. Both shops plan to run annual discount sales in the last week of December. For this, they want to attract more number of customers by using advertisements through leaflets, newspaper, radio and television. By seeing the market trend, the shop *BATA* constructed the following payoff matrix, where the numbers in the matrix indicate a gain or a loss of customers:

***DSI***

| | | *Leaflets* | *Newspaper* | *Radio* | *Television* |
|---|---|---|---|---|---|
| ***BATA*** | *Leaflets* | 20 | -20 | 40 | 10 |
| | *Newspaper* | 60 | 30 | 120 | 40 |
| | *Radio* | -30 | 20 | 10 | 60 |
| | *Television* | 20 | -30 | 70 | 70 |

(i) Is there a saddle point of the game? Justify your answer.

(ii) Solve the game and find the value of the game.

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**Question 02**

(a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point.

*Player B*

*Player A*

| | $B_1$ | $B_2$ |
|---|---|---|
| $A_1$ | $a$ | $-b$ |
| $A_2$ | $-c$ | $d$ |

where $a, b, c, d$ are all non-negative.

Prove that the optimal strategies are:

$$A = \begin{bmatrix} A_1 & A_2 \\ \dfrac{c+d}{a+b+c+d} & \dfrac{a+b}{a+b+c+d} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & B_2 \\ \dfrac{b+d}{a+b+c+d} & \dfrac{a+c}{a+b+c+d} \end{bmatrix}$$

and

the value of the game $v = \dfrac{ad - bc}{a+b+c+d}$.

(b) Player *I* holds a black Ace and a red 8 from a standard 52 card deck. Player *II* holds a red 2 and a black 7. The players simultaneously choose a card to play. If the chosen cards are of the same color, Player *I* wins. Player *II* wins if the cards are of different colors. The amount won is a number of rupees equal to the number on the winner's card (Ace counts as 1).

(i) Construct the payoff matrix with respect to the player *I*.

(ii) Is there a saddle point? Justify your answer.

(iii) Find the value of the game and the optimal mixed strategies of the players.

**Question 03**

(a) Explain characteristics and classifications of queuing models.

(b) A branch of a Bank has only one typist. Since the typing work varies in length (number of pages to be typed), the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letters arrive at a rate of 5 per hour during the entire 8-hour work day. If the type writer is valued at Rs. 15 per hour, determine

(i) probability of the typist being idle,

(ii) probability that there are at least 3 letters in the system

(iii) the percent time an arriving letter has to wait,

(iv) the average system time,

(v) the average idle time cost of the typewriter per day.

**Question 04**

An insurance company has 3 claim adjusters in its main office. Customers are found to arrive in Poisson distribution at the rate of 5 per hour for settling claims against the company. The service time is found to have an exponential distribution with mean 25 minutes. Claimants are processed on first come first served basis. Calculate

(a) the average number of customers in the system,

(b) the average time a customer spends in the system,

(c) the average queue length,

(d) the average waiting time for customers,

(e) the number of hours per week spent on performing the job,

(f) the probability that at least a claim adjuster is waiting for the customer,

(g) the expected number of idle time claim adjusters at any specific moment.

**Question 05**

(a) Define the term "inventory".

(b) What are the advantages and disadvantages of having inventories?

(c) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and instantaneous supply.

(d) An industry estimates that it will sell 12,000 units of its product for the forthcoming year. The ordering cost is Rs.100 per order and the carrying cost per unit per year is 20% of purchase price per unit. The purchase price per unit is Rs.50. Find

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(i) the Economic Order Quantity,

(ii) the number of orders per year,

(iii) the time between successive orders,

(iv) the total cost per year.

**Question 06**

(a) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and replenishment rate is finite.

(b) A unit is used at the rate of 100 per day and can be manufactured at a rate of 600 per day. It costs Rs. 2000 to set up the manufacturing process and holding cost is Rs. 0.1 per unit per day. Shortage is not allowed. Find the minimum cost and the optimum number of units per manufacturing run.

Using the formula established in part(a), answer the following questions:

(i) find the optimum number of units per manufacturing run,

(ii) find the time of cycle,

(iii) determine the minimum total cost per manufacturing run.

## Formulas (in the usual notation)

### (M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq \text{n}) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu-\lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad E(v) = \frac{1}{\mu-\lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

### (M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1-N\rho^{N-1}+(N-1)\rho^N\right]}{(1-\rho)\left(1-\rho^{N+1}\right)}$$

$$E(n) = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)\left(1-\rho^{N+1}\right)} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

### (M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ;1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ;n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1} \qquad E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

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**(M/M/C): (N/FIFO) Model**

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;0 \le n \le C \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;C < n \le N \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=C}^{\infty} \frac{1}{C^{n-C}C!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {\left[E(n)\right]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

**(M/M/R):(K/GD) Model**

$$P_n = \begin{cases} \binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ 0 \le n < R \\ \binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n P_0 & ;\ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda\left[K - E(n)\right]}$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda\left[K - E(n)\right]}$$

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