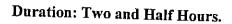
The Open University of Sri Lanka
B.Sc./B.Ed. Degree Continuing Education Programme
Final Examination-2008/2009
PMU 2192/ PME 4192- Linear Algebra
Pure Mathematics



Date: 08-01-2009.

Time: 09.30 a.m. - 12.00 noon

## Answer FOUR questions only.

- (1) (i) Define each of the following:
  - (a) a spanning set,
  - (b) a subspace,
  - (c) a basis.
  - (ii) Give an example for a spanning set which is not a basis.
  - (iii) Let  $W = \{(a, b, c, d) | a + b = 0, c = 2d\}$ . Show that W is a subspace of  $\mathbb{R}^4$ . Find a basis for W and the dimension of W.
- (2) (i) Let S be any finite subset of the vector space  $\mathbb{R}^3$ . Prove that the span of S is a subspace of  $\mathbb{R}^3$ .
  - (ii) Determine whether each of the following sets of vectors is linearly independent or dependent:
    - (a)  $\{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$  in  $\mathbb{R}^3$ .
    - (b)  $\{(1, 2, -1, 1), (-3, 1, 2, -1), (-3, 8, 1, 1)\}$  in  $\mathbb{R}^4$ .
    - (c)  $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$  in  $\mathbb{R}^3$ .
  - (iii) Which of the following are linear transformations? Justify your answer.
    - (a)  $T: \mathbb{R}^2 \to \mathbb{R}^3$

$$T(x, y) = (x + 2y, x - y, y)$$

(b) 
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
  
 $T(x, y, z) = [x + y, y, (x + z)^2]$ 

(c) 
$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
  
 $T(x, y) = (2x + 5y, 0)$ 

- (3) (i) Define an inner product space.
  - (ii) Find  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$  if the set  $\left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}$  is an orthonormal set in  $\mathbb{R}^3$ .
  - (iii)  $\{1, x, x^2, x^3\}$  is a linearly independent set of continuous functions on the interval [-1, 1]. Find the corresponding othonormal set starting from the given set.
- (4) (i) Find the rank of the matrix  $\begin{pmatrix}
  0 & c & -b & a' \\
  -c & 0 & a & b' \\
  b & -a & 0 & c' \\
  -a' & -b' & -c' & 0
  \end{pmatrix}$  where a, b and c are positive and aa' + bb' + cc' = 0.

(ii) Let 
$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & -3 & 4 \\ 4 & 2 & 1 \end{pmatrix}$$
. Find the inverse of  $A$ .

(iii) Let 
$$A = \begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{pmatrix}$$
. Find non-singular matrices  $P$  and  $Q$  such that

PAQ is of the normal form.

Hence determine the rank of A.

- (5) (i) If A is a square matrix of order n and rank (n-1), show that Adj A is not equal to the null matrix.
  - (ii) Show that if two matrices A and B have the same order and same rank, then there exist non-singular matrices R and S such that B = RAS.
  - (iii) Solve the following homogeneous linear equations in x, y, z, t given that a, b, c are all distinct.

$$(b-a)y + (c-a)z + (b+c)t = 0$$

$$(a-b)x + (c-b)z + (c+a)t = 0$$

$$(a-c)x + (b-c)y + (a+b)t = 0$$

$$(b+c)x + (c+a)y + (a+b)z = 0.$$

- (6) (i) If k is a non-zero scalar, prove that the characteristic roots of kA are k times the characteristic roots of A.
  - (ii) Let  $A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ . Using the Cayley Hamilton Theorem, express  $(5A^5 3A^4 + A^2 5I)$  in the form  $\alpha A + \beta I$ , where  $\alpha$  and  $\beta$  are to be determined.
  - (iii) Let  $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ . Find an orthogonal matrix P such that

P'AP is a diagonal matrix, where P' is the transpose of P.