The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

No Book Test 2015/2016

Level 04 Applied Mathematics

APU 2140/APE 5140– Statistical Distribution Theory

Duration: - One hour

Date: - 14-05-2016 **Time: - 9.00 a.m. - 10.00 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.

Answer all questions.

1.

Particular academic program has three levels and the average marks of ***level 1***, ***level 2*** and ***level 3*** of a student are given by the variables X_1, X_2 and X_3 respectively. The ***final average*** of a student is calculated by the formula.

$$\textit{final average} = \frac{X_1 + 2X_2 + 3X_3}{6}.$$

From the past experience it is known that $X_1 \sim N(50,100)$ $X_2 \sim N(45,100)$ and $X_3 \sim N(40,100)$. Assuming that X_1, X_2 and X_3 are independent:

(i) Calculate the expected ***final average*** of a student who enrolled in the above academic programme.

(ii) Calculate the variance of ***final average*** of a student who enrolled in the above academic programme.

(iii) Write the distribution of ***final average***. Clearly state the values of any parameters involved.

(iv) Find the probability of a student obtaining a ***final average*** in between 40 and 60.

(v) Suppose that 750 students are enrolled to the above programme in 2016 batch. According to the rules and regulations of the above programme, student has to get a minimum of 40 for the ***final average***, for successful completion of the programme. . Calculate the expected number of students who will complete the course in 2016 batch relying on the past experience.

2.

(a) Random variable X has the probability mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0,1,2,3, \dots\dots\dots$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i), show that $E(X) = \lambda$ and $Var(X) = \lambda$

(b) Suppose $Y_1 \sim exp(5)$ $Y_2 \sim exp(5)$ **and** $Y_3 \sim exp(5)$. Assume all these variables are independent. Calculate $Pr(Y_1 + Y_2 + Y_3 \leq 1)$

(c) In the case of snake bites, often, lethal doses are not imparted. Suppose 80% of the snake bite cases recover even without treatment. What is the probability that out of 200 cases untreated more than 190 will recover.

Left tail values of Standard Gamma Table

W - gamma(α,1)}

This table contain the probabilities Pr(W ≤ w)

	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432