00135

The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
FINAL EXAMINATION 2014/2015
Level 04 Applied Mathematics
APU 2140– Statistical Distribution Theory

Duration: - Two Hours.

DATE: 11-05-2015 **Time: 9.30 a.m. – 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.
Answer four questions only.

(1) A company that produces a certain electrical product claims that the life time X (years) is a random variable with density function

$$f(x) = \begin{cases} x^2 & ; \quad 0 \leq x < 1 \\ \frac{7-3x}{4} & ; \quad 1 \leq x < \frac{7}{3} \\ 0 & ; otherwise \end{cases}$$

(i) Find the expected life time and variance of a randomly selected electrical product.
(ii) Find the cumulative distribution function of ***X.***
(iii) Find the probability that a randomly selected product will not fail within two years.
(iv) Find the median lifetime of a randomly selected product.

(2)

(a) Random variable X has the probability mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0,1,2,3,\dots\dots\dots$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i), show that $E(X) = \lambda$ and $Var(X) = \lambda$

(b) Suppose the number of babies born during an 8-hour shift at a hospital's maternity wing follows the distribution given in part (a) with a mean of 16.

(i) Find the probability that three babies are born during a particular 1-hour period in this maternity wing.

(ii) Suggest another distribution that can be used to approximate the number of babies born during an 8-hour shift. Clearly sate the distribution and the parameters that you suggest. Using the distribution you proposed, find the probability that minimum of 14 babies are born during a particular 8- hour session in this maternity wing.

(3)

(a) An inspector is looking for the non conforming welds in a pipe line. The probability that any particular weld is defective is 0.01. The inspector is determined to keep walking until he finds three defective welds. If the first weld is located at a distance of 20m apart from the starting of the pipe line and the welds are located in 20m apart,

(i) Find the probability that the inspector will have to walk 1km.

(ii) Find the probability that the inspector will have to walk more than 1km.

(iii) State the assumptions that you have made in parts (i) & (ii).

(b) In the case of snake bites, often, lethal doses are not imparted. Suppose 80% of the snake bite cases recover even without treatment. What is the probability that out of 100 cases untreated more than 85 will survive.

(4)

Life time of a light bulb manufactured by ABC Company is normally distributed with mean 300 days and a standard deviation of 50 days.

(i) What is the probability that a light bulb will last at most 365 days?

(ii) What is the probability that a light bulb fails in between 200 days and 400 days?

(iii) The production manager of the ABC Company plans to set a warranty period (in days) such that 75% of the bulbs should not fail during the warranty period. Calculate the warranty period.

(iv) Random sample of 10 bulbs from the above population were tested and sample mean $\bar{X}$ was estimated. Find the probability that sample mean of lifetimes exceeds 325 days.

(5)

(a) Particular academic program has three levels and the average marks of ***level 1***, ***level 2*** and ***level 3*** of a student are given by the variables X_1, X_2 and X_3 respectively. The ***final average*** of a student is calculated by the formula

$$final\ average = \frac{X_1 + 2X_2 + 3X_3}{6}.$$

From the past experience it is known that $X_1 \sim N(50,100)$ $X_2 \sim N(45,100)$ and $X_3 \sim N(40,100)$. Suppose that 750 students are enrolled to the above programme in 2015 batch. According to the rules and regulations of the above programme, student has to get a minimum of 40 for the ***final average***, to complete the programme successfully.

Assuming that X_1, X_2 and X_3 are independent calculate the expected number of students who will complete the course in 2015 batch relying on to the past experience.

(b) Suppose $Y_1 \sim exp(5)$ $Y_2 \sim exp(5)$ $Y_3 \sim exp(5)$. Assume all these variables are independent. Calculate $\Pr(Y_1 + Y_2 + Y_3 \leq 1)$

(6)

Four white balls and three black balls are distributed in two urns in such a way that the first urn contains two white balls and one black ball. In a game, a ball is drawn randomly from the first urn and then placed it in the second urn. Then a ball is drawn randomly from the second urn and placed it in the first urn. This concludes the game. Let X denotes the number of white balls in the first urn after the game. Let Y denotes the number of white balls in the second urn after the game.

(i) Find the possible values for X and Y.

(ii) Find the marginal probability mass functions of X, and Y.

(iii) Find the joint probability density function of X and Y.

(iv) Find the distribution of X|Y=1 and hence calculate E(X|Y=1).

(v) Calculate E(XY)

Left tail values of Standard Gamma Table

W - gamma(α,1)

This table contain the probabilities Pr(W ≤ w)

w	α					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432