The Open University of Sri Lanka
B.Sc/B.Ed. Degree, Continuing Education Programme
Final Examination - 2013/2014
Level 04 Applied Mathematics
APU 2140– Statistical Distribution Theory

Duration: - Two Hours.

DATE: - 17-06-2014. Time: - 9.30 a.m. - 11.30 a.m.

Non programmable calculators are permitted. Statistical tables are provided.

Answer four questions only

(1)

The probability density function of the continuous random variable X is

$$f_X(x)=\begin{cases} \dfrac{x}{k} & when\ 0<x<3 \\ \dfrac{6-x}{k} & when\ 3\le x<6 \\ 0 & otherwise \end{cases}$$

(i) Find the value of k

(ii) Find $Pr(-2\le X\le 5)$

(iii) Find the mean and the variance of the random variable X.

(iv) Let $Y=3X+1$. Find the mean and the variance of Y.

(v) Find $Pr(Y\le 3)$.

(2)

A company that produces a certain pain killer claims that the time X (in minutes) to relieve a pain after a tablet of their pain killer has the cumulative distribution function

$$F_X(x) = 1 - e^{-kx};\ x > 0$$

Median time to relieve a pain after taking a tablet is five minutes.

(i) Find the value k.

(ii) Derive the density function of X.

(iii) Show that the moment generating function of X is

$$M_X(t) = \left(1 - \frac{t}{k}\right)^{-1};\ t < k$$

Hence find the mean time to relieve a pain after taking a tablet.

(iv) What is the probability that a randomly selected patient with a pain will get relief within 6 minutes after taking a tablet?

(v) 25% of patients with pain will not get relief within t minutes after taking a tablet of their pain killer, find the value of t.

(3)

(a) According to the past data, the probability of raining to a stadium in a certain area on a given day during December is 0.05. A cricket match is going to be held in this stadium from 15th December 2014 to 19th December 2011.

(i) What is the probability that there will be rain only on the last day of the match?

(ii) Find the probability that there will be no rain on the second, third and fourth days of the tournament.

(iii) State the assumptions if any that you have made for answering above parts (i) and (ii)

(b) Suppose $X_1 \sim N(5,9)$ $X_2 \sim N(3,4)$ $X_3 \sim \exp(5)$ $X_4 \sim \exp(5)$ $X_5 \sim \exp(5)$
Assume all these variables are independent. Calculate the following probabilities.

(i) $\Pr((\frac{X_1-5}{3})^2 \leq 0.1)$

(ii) $\Pr(X_3 + X_4 + X_5 \leq 1)$

(4)

(a) A restaurant kitchen has two food mixing machines A and B. The average number of times A brakes down per week is 0.3 and the average number of times B brakes down per week is 0.2. Find the probability that more than 4 total break downs of both machines per month assuming that month has four weeks.

(b) A factory packs bulbs in boxes of 200. The probability of a bulb of poor quality is 0.008. What is the probability that box contains 5 bulbs of poor quality.

(c) The number of bacteria on a plant follows a Poisson distribution with parameter 20. Find the probability of there being 10 to 30 bacteria on a plant.

(5)

The pair of random variables X and Y has the following desecrate joint distribution.

P(X,Y)		***X***		
		0	1	2
Y	0	0.03	***a***	0.12
	1	0.05	0.15	0.16
	2	0.02	0.2	0.12

(i) Find the constant a.

(ii) Calculate the following probabilities

(I) $Pr(X > 1, Y > 1)$ *(II)* $Pr(X=2, Y \leq 1)$

(iii) Find the marginal probability distribution of X.

(iv) Are X and Y independent? Justify your answer.

(v) Find the probability distribution of X given $Y \leq 1$.

(vi) Find the expected value of X given $Y \leq 1$.

(6)

(a) The lifetime of an automotive battery is normally distributed with mean 48 months and standard deviation 9 months.

(i) Find the probability that lifetime of a randomly selected battery is in between 39 months and 57 months.

(ii) Find the probability that lifetime of a randomly selected battery exceeds five years.

(iii) In setting warranties on batteries, the manufacturer would like to set the expiration time of the warranty at such a level that 95% of batteries made will remain in working condition throughout the warranty period. What should be the warranty period?

(b) A machine is set to produce ball-bearings with mean diameter 1.2cm. Each day a random sample of 50 ball-bearings are selected and the diameters are accurately measured. If the sample mean diameter lies outside the range 1.18cm to 1.22cm then it will be taken as evidence that mean diameter of the ball bearing produced is not 1.2 cm. The machine will then be stopped and adjustments are made to it. Assume that the diameter has a standard deviation of 0.075 cm,

(i) Find the distribution of sample mean diameter. Clearly sate the distribution and the necessary parameters.

(ii) Dose your answers for (i) depend on the central limit theorem? Give reasons for your answer.

Left tail values of Standard Gamma Table

W - gamma(α,1)}

This table contain the probabilities Pr(W ≤ w)

w	α					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432