The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
FINAL EXAMINATION 2010/2011
Level 04 Applied Mathematics
APU 2140– Statistical Distribution Theory

Duration: - Two Hours.

DATE: - 29-12-2010. Time: - 9.30 a.m. – 11.30 a.m.

Answer FOUR questions only.

Non programmable calculators are permitted.

(1) Suppose X is a random variable with the density function $f_X(x) = \lambda e^{-\lambda x}$; $x > 0, \lambda > 0$

(i) Show that the moment generating function of X is given by $\frac{\lambda}{\lambda - t}$; $t < \lambda$

(ii) Using the moment generating function, show that $E(X) = \frac{1}{\lambda}$ and $Var(X) = \frac{1}{\lambda^2}$

(iii) Find $E(2X + 3)$ and $Var(2X + 3)$ in terms of λ

(iv) Find the cumulative distribution function of X.

(v) Let $\lambda = 0.14$ Find $\Pr(X \le 5)$

(2) The ABC Company has two showrooms in Colombo city limits. One is located at Petta and the other one located at Maradana. Both of these showrooms sell Sun brand electric fans. Let X be the number of Sun brand electric fans sold per day at the Petta showroom and let Y be the number of Sun brand electric fans sold per day at the Maradana showroom. The following table shows the joint probabilities, according to the past data.

P(x,y)		*x*		
		0	1	2
y	0	0.03	0.15	0.12
	1	0.05	0.15	0.16
	2	0.02	0.2	0.12

(i) Find the marginal distribution functions of X and Y.

(ii) The sales manager of the ABC Company claims that the sales of Sun brand electric fans at Petta and Maradana showrooms are independent. Do you agree with the sales manager's claim? Justify your answer.

(iii) Find the total expected sales of Sun brand fans per day at two showrooms.

(iv) Assume that these two showrooms open at 9.00a.m. and close at 4.00 p.m. on week days. On a particular weekday salesman of the Maradana showroom has sold their first Sun brand electric fan at 1.00 p.m. What is the expected sales of Sun brand fans at Petta showroom on that day.

(3) Kamal is a high school basketball player. He is a 70% free throw shooter. That means his probability of making a free throw shoot is 0.70. During the season

(i) What is the probability that Kamal makes minimum of four free throw shoots out of ten shots?

(ii) What is the probability that Kamal makes his third free throw shoot on his sixth shot?

(iii) What is the probability that Kamal makes his first free throw shoot on his fifth shot?

State all the assumptions you have made under the (i) , (ii) and (iii)

(4) Life time of a light bulb manufactured by ABC Company is normally distributed with mean 300 days and a standard deviation of 50 days.

(i) What is the probability that a light bulb will last at most 365 days?

(ii) What is the probability that life time of the light bulb exceeds 400 days?

(iii) The production manager of the ABC Company plans to set a warranty period (in days) such that 75% of the bulbs should not fail during the warranty period. Calculate the warranty period.

(5) Suppose $X_1 \sim N(5,9)$ $X_2 \sim N(3,4)$ $X_3 \sim exp(5)$ $X_4 \sim exp(5)$ $X_5 \sim exp(5)$

Assume all these variables are independent. Calculate the following probabilities.

(i) $Pr(2X_1 + 3X_2 \geq 20)$

(ii) $Pr((\frac{X_1-5}{3})^2 \leq 0.1)$

(iii) $Pr(X_3 + X_4 + X_5 \leq 1)$

6)

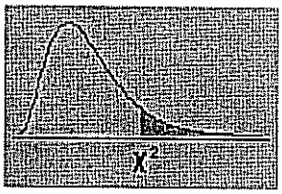
(a) Describe the central limit theorem in your own words.

(b) A random sample of 3 beetles is drawn from a population of beetles whose length X is normally distributed with mean 2.4cm and standard deviation of 0.36 cm. Let $\overline{X}$ be the mean of the lengths of the 4 beetles. State the distribution of $\overline{X}$, giving the values of its parameters. Does your answer depend on the central limit theorem? Justify.

(c) A machine is set to produce ball-bearings with mean diameter 1.4cm. Each day a random sample of 50 ball-bearings are selected and the diameters are accurately measured. If the sample mean diameter lies outside the range 1.38cm to 1.42cm then it will be taken as evidence that mean diameter of the ball bearing produced is not 1.4 cm. The machine will then be stopped and adjustments made to it. Assuming that the diameter has a standard deviation of 0.1 cm, find the probability of the machine being stopped unnecessarily for adjustments.

Does your answer depend on the central limit theorem? Justify.

Right tail areas for the *Chi-square* Distribution



df\area	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960

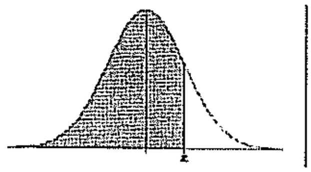
Left tail values of Standard Gamma Table

W - gamma(α,1)}

This table contain the probabilities Pr(W ≤ w)

	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432

Z table – Left tail values



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890