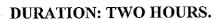
The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Final Examination – 2017/2018

Applied Mathematics - Level 04

ADU4303/APU2144 - Applied Linear Algebra and Differential Equations



Date: 18.04.2019 Time: 9.30am-11.30am

ANSWER FOUR QUESTIONS ONLY.

1. (i) Find the determinant of A where

$$A = \begin{pmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \alpha) & 1 \end{pmatrix}$$

(ii) Determine the rank of each of the following matrices:

(a)
$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{pmatrix}$$
 (b)
$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

- (iii) Determine the values of λ and μ such that the following system of equations has
 - (a) no solution
 - (b) a unique solution
 - (c) infinitely many solutions.

$$2x-5y+2z = 8$$
$$2x+4y+6z = 5$$
$$x+2y+\lambda z = \mu$$

Justify your answers.

2. (i) Let
$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$
. Use the Cayley-Hamilton theorem to find A^{-1} and A^{4} .

- (ii) Transform the quadratic form $10x_1^2 + x_2^2 + x_3^2 6x_1x_2 2x_2x_3 + x_3x_1$ to a canonical form, using an orthogonal transformation.
- 3. Solve each of the following systems of differential equations:

(i)
$$\dot{x}_1 = x_1 + 3x_2 - 3x_3$$

$$\dot{x}_2 = -3x_1 + 7x_2 - 3x_3$$

$$\dot{x}_3 = -6x_1 + 6x_2 - 2x_3$$

(ii)
$$\dot{x}_1 = -10x_1 + 6x_2 + 10e^{-3t}$$

$$\dot{x}_2 = -12x_1 + 7x_2 - 18e^{-3t}$$

(iii)
$$\ddot{x}_1 = x_2$$

$$\ddot{x}_2 = x_1$$

In the above, the dots represent the derivatives with respect to a parameter t.

4. (i) Find a sinusoidal particular solution for the following system of differential equations:

$$\ddot{x}_1 + 3\dot{x}_2 + 2x_1 = \sin 3t$$

$$\ddot{x}_2 + \dot{x}_1 = \cos 3t$$

(ii) Find the general solution of the differential equation

$$x^{2} \frac{d^{2} y}{dx^{2}} - 5x \frac{dy}{dx} + 8y = 2x^{3}.$$

(iii) Suppose that u is a function of two variables x and t, satisfying the partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$, where c is a non-zero constant.

By transforming to new variables ζ and ϕ , where $\zeta = x - ct$ and $\phi = x + ct$, show that the equation can be simplified to $\frac{\partial^2 u}{\partial \zeta \partial \phi} = 0$. Hence find the general solution of the partial differential equation in terms of x and t.

5. (i) Find the equations of the characteristic curves for the following partial differential equation and determine the new variable ϕ .

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 2.$$

- (ii) Hence obtain the general solution u(x, y) of the partial differential equation given in part (i).
- (iii) Find the particular solution which satisfies the condition $u(x, y) = \sin x$ on y = -x.



6. (i) Show that the general solution of the equation

$$y^{2} \frac{\partial^{2} u}{\partial x^{2}} - 2xy \frac{\partial^{2} u}{\partial x \partial y} + x^{2} \frac{\partial^{2} u}{\partial y^{2}} - \frac{y^{2}}{x} \frac{\partial u}{\partial x} - \frac{x^{2}}{y} \frac{\partial u}{\partial y} = 0, \quad (x \neq 0, \ y \neq 0)$$

may be found by reducing it to the standard form $\frac{\partial^2 u}{\partial \phi^2} - \frac{1}{\phi} \frac{\partial u}{\partial \phi} = 0$,

where
$$\zeta = x^2 + y^2$$
 and $\phi = y$.

Use the method of characteristics to derive these expressions for ζ and ϕ . Hence find the general solution of the partial differential equation.

(ii) Show that the equation

$$(x^{2} - 1)\frac{\partial^{2} u}{\partial x^{2}} + 2y\frac{\partial^{2} u}{\partial x \partial y} - \frac{\partial^{2} u}{\partial y^{2}} = 0$$

- is (a) hyperbolic for all (x, y) outside the region R bounded by the circle $x^2 + y^2 = 1$,
 - (b) parabolic on the boundary of the region R and
 - (c) elliptic for all (x, y) inside R.