AC / systems -

00031 (99)

THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
AMU 3189/ AME 5189- STATISTICS II
OPEN BOOK TEST - 2010/2011

Duration: One and Half Hours.

Date: 08.03.2011 **Time: 4.00 p.m. – 5.30 p.m.**

Non programmable calculators are permitted.

Answer all questions.

(1) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{\theta} \quad ; \quad 0 \le x \le \theta$$

(i) Find the mean of the above distribution

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you an unbiased estimator for θ? Prove your answer.

(iii) Derive the maximum likelihood estimator for θ.

(iv) A sample drawn from the above distribution is given in the following table. Calculate the moment and maximum likelihood estimator based on the part (ii) and (iii).

0.92	0.57	1.51	4.75	2.27
1.57	4.12	1.9	0.19	0.82
0.25	3.58	2.51	3.97	3.81
4.45	2.32	1.27	0.72	3.02

(2)

(a) Describe what is meant by the statement that the *density function* $f(x;\theta)$ *belong to the exponential family.* Assume that Θ is a single parameter.

(b) State whether or not the following density function belongs to the exponential family. Prove your answer.

$$f(x;\theta) = \begin{cases} {n_{c_x}}\, \theta^x (1-\theta)^{n-x} & ; \ x = 1,2,3,\ldots\ldots, 0 \le \theta \le 1 \\ 0 & ; \quad otherwise \end{cases}$$

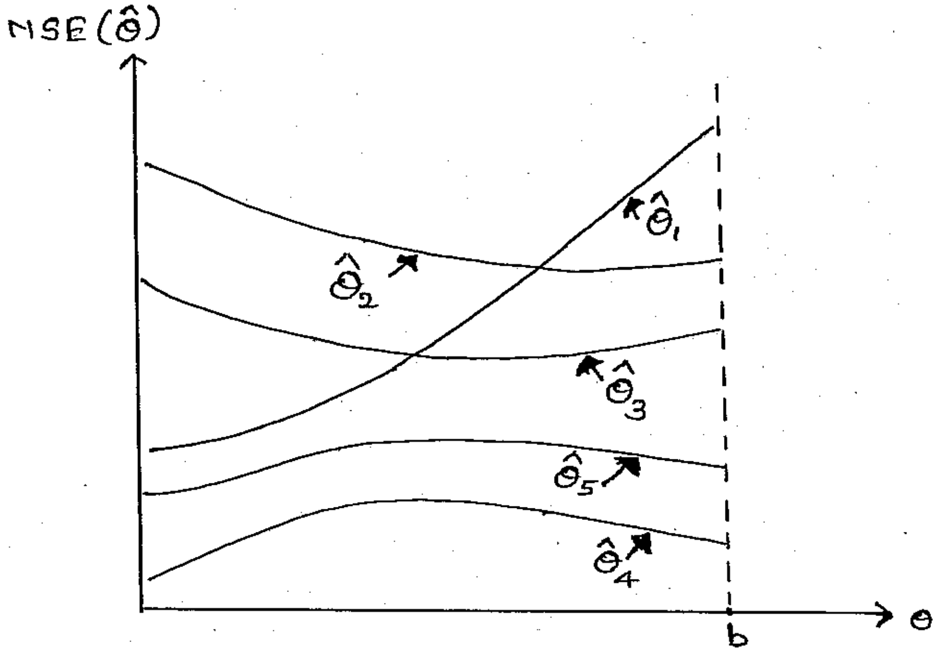
(c) For the parameter θ in the above density function given in part (b), obtain a sufficient statistic for θ. Hence or otherwise obtain a uniformly minimum variance unbiased estimator (UMVUE) for θ in part (b).

(3) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a distribution with density given by

$$f(x;\theta);\ -\infty \le x \le \infty, \qquad 0 \le \theta \le b$$

Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5$ are functions of $X_1, X_2, X_3, \ldots\ldots\ldots X_n$.

Suppose $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_5$ are unbiased estimators for parameter θ; $\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5$ are sufficient statistics for parameter θ and $\hat{\theta}_3$ is an asymptotically unbiased estimator for parameter θ. Following graph gives the mean square error of above statistics.



(a)

(i) A student says that estimator $\hat{\theta}_3$ is better than estimator $\hat{\theta}_2$. Do you agree with the student. Justify your answer.

(ii) Among the statistics $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5$ which one is the best statistic for θ. Justify your answer.

(b) Which of the following statistics are unbiased estimators for θ. Prove your answer.

(i) $\frac{2\widehat{\theta_1}+\widehat{\theta_2}+\widehat{\theta_5}}{4}$

(ii) $\frac{2\widehat{\theta_1}+\widehat{\theta_2}-\widehat{\theta_5}}{2}$

(iii) $\frac{\widehat{\theta_1}+\widehat{\theta_4}}{2}$

(c) Prove that variance$(\widehat{\theta}_2) \ge$ variance$(\widehat{\theta}_5)$