

## The Open University of Sri Lanka

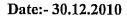
## B.Sc Degree Programme/ Continuing Education Programme

Final Examination- 2010/2011

Level 04- Pure Mathematics

PMU2191/PME4191 - Vector Analysis

**Duration**:- Two hours



Time:- 1.00p.m.-3.00p.m.

## Answer Four Questions only.

- 1. (a) Consider the equation  $\left(p + \frac{a}{v^2}\right)(v b) = ct$ , where the variables p, v and t denote the pressure, volume and temperature respectively, and a, b, c are constants. Show that  $\left(\frac{\partial p}{\partial v}\right)\left(\frac{\partial v}{\partial t}\right)\left(\frac{\partial t}{\partial p}\right) = -1$ .
  - (b) Find the Second order Taylor polynomial for the function  $f(x, y) = e^x \sin ay$  about the point (0, 0).
  - (c) Considering a suitable multivariable function, estimate the value of  $2.23\sqrt{5.36^3-1.96^2}$ .
- 2. (a) (i) Define a stationary point of a single valued function f(x, y), defined over the domain D. Explain briefly how you would determine its nature.
  - (ii) Determine the nature of the stationary points of the function  $f(x, y) = 3x^2y + y^3 3x^2 3y^2 + 2.$
  - (b) Find the directional derivative of the function  $\phi = x^2yz + 4xz^2$  at the point (1, -2, -1) in the direction of the vector  $2\underline{i} j 2\underline{k}$ .
  - (c) Evaluate the scalar line integral of the vector function  $\underline{F} = (3x^2 4xy)\underline{i} + (3y^2 5xy)\underline{j}$  along the path C, where C is the arc of parabola  $y = x^2$  from A = (0, 0) to B = (2, 4).

- 3. (a) Use surface integrals to find the closed area bounded by  $y = x^2 4x + 5$  and  $y = -x^2 + 4x 1$ . (No credit will be given for other methods)
  - (b) Evaluate the surface integral  $\int_{S} y^2 dA$ , where S is the boundary of the triangular region enclosed by y=1, x=2 and y=x+1.
  - (c) Use surface integrals to calculate the area of the quarter of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  for which  $x \ge 0$  and  $y \ge 0$ .
- 4. (a) Find the moment of inertia of a solid circular cylinder of constant density  $\rho$ , radius a and height h about its axis of symmetry.
  - (b) Find  $\int_{B} z^{3} \sqrt{x^{2} + y^{2} + z^{2}} dV$ , where B is the solid hemisphere, with the centre at the origin, radius 1, that lies above xy plane.
  - (c) Find volume of the tetrahedron bounded by the coordinate planes and the plane 3x + 2y + 6z = 6.
- 5. (a) State Gauss' divergence theorem.
  - (b) Use Gauss' divergence theorem to evaluate the surface integral  $\oint_S gradf \cdot \underline{n} \, dA$ , where f is the scalar field  $f(x, y, z) = x^4 + y^4 + z^4$  and S is the spherical surface  $x^2 + y^2 + z^2 = a^2$ , the vector  $\underline{n}$  having its usual meaning.
  - (c) Prove that (i)  $\nabla \cdot \left( r \nabla \left( \frac{1}{r^3} \right) \right) = 0$  (ii)  $\nabla \cdot \left( r^3 \underline{r} \right) = 6r^3$ , where  $\underline{r}$  and  $\underline{r}$  carry the usual meanings.
- 6. (a) State Stoke's theorem.
  - (b) Verify the Stoke's theorem for the vector field  $\underline{F} = -y\underline{i} + x\underline{j}$ , where the surface S is the disk  $x^2 + y^2 \le a^2$  in the z = 0 plane, oriented by the upward unit normal  $\underline{n}$ .
  - (c) If  $\underline{v} = \underline{w} \times \underline{r}$ , prove that  $\underline{w} = \frac{1}{2} curl \underline{v}$ , where  $\underline{w}$  is a constant vector and  $\underline{r}$  carries the usual meaning.