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The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Final Examination 2017/2018

Level 04 Applied Mathematics

ADU4300/ADE4300– Statistical Distribution Theory - APU2140

Duration: - Two hours

Date: - 27-09-2018 **Time: 1.30 – 3.30pm**

Non programmable calculators are permitted. Statistical tables are provided.

Answer four questions only.

1.

A manufacturer of electronic calculators offers a warranty. If the calculator fails for any reason during this period, it is replaced. The time to fail *(X)* in years is well modeled by the following probability distribution.

$$f(x) = 0.125e^{-0.125x} \quad ; x > 0$$

(i) If the manufacturer of electronic calculators offers a three years warranty period, what percentages of the calculators will fail within the warranty period?

(ii) Find the cumulative distribution function of the time to failure of a randomly selected calculator.

(iii) Determine the probability that a calculator fails in the interval from 6 to 9 years.

(iv) Determine the number of years that 10% of the calculators will be failed in a batch of production.

(v) Find the expected time to fail a calculator selected randomly.

2.

(a) A random Variable X has the mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0, 1, 2, 3, \ldots\ldots\ldots \text{ and } \lambda > 0$$

Let $M_x(t)$ be the moment generating function of X.

(i) Show that $M_x(t) = e^{\lambda(e^t - 1)}$

(ii) Using part (i), show that $E(X) = \lambda$ and $Var(X) = \lambda$

(b) Suppose the number of babies born during an 8-hour shift at a hospital's maternity wing follows the distribution given in part (a) with a mean of 16.

(i) Find the probability that three babies are born during a particular 1-hour period in this maternity wing.

(ii) Give another distribution which can be used to approximate the number of babies born during an 8-hour shift. Clearly sate the distribution and the parameters of the distribution that you suggest. Hence, find the probability that minimum of 14 babies are born during a particular 8- hour session in this maternity wing.

3.

In a quality control process, sample of 30 parts from a metal punching process are selected every hour. When the metal punching process is in control 1% of the parts require rework. Let X denotes the number of parts in the sample of 30 that require rework.

The metal punching process is stopped for adjustments if $\mathbf{X > a + 3b}$

Where $\boldsymbol{a = E(X)}$ ***when punching process is in control*** and

$\boldsymbol{b = \sqrt{Var(X)}}$ ***when punching process is in control*** .

(i) Suggest a suitable probability distribution for X? Clearly state the mass function and the parameters of the distribution.

(ii) Find the probability that two parts in the selected sample require rework in a particular hour, given that the process is in control.

(iii) Find the expected number of parts that will require rework in the sample when the process is in control.

(iv) Find the standard deviation of the number of parts in the sample that require rework when the process is in control.

(v) Find the probability of punching process being stopped for adjustments when the process is in control.

(vi) Find the probability of process not being stopped for adjustments when 2% of the parts require rework.

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4.

(a) A machine produces components of mean and standard deviation of the diameter are 1.35 cm and 0.05 cm. The diameters are assumed to be normally distributed. Suppose all components with diameters outside the range 1.25 to 1.45 are considered as defective items and are rejected.

(i) What proportion of components will be rejected in a batch of production? Give your answer rounded to the second decimal place.

(ii) Suppose that production items will be checked one by one.

I. Find the probability that first defective item found is the 10th checked item.

II. Find the probability that third defective item that found is the 15th checked item

(b) Suppose that X_1, X_2, X_3, X_4 are independent random variables described as

$$X_1 \sim N(3,4) \qquad X_2 \sim N(5,9) \qquad X_3 \sim exp(3) \qquad X_4 \sim gamma(3,3)$$

Find the following probabilities. Show your calculations and state the justifications clearly. You may use the gamma table given at the end of the paper wherever necessary.

(i) $Pr[(2X_1 + 3X_2) < 25\ )]$

(ii) $Pr[(X_3 + X_4) > 3\ )]$

5.

The joint distribution of random variables X and Y is given below.

$P(X=x, Y=y)$		X		
		0	1	2
Y	1	0.12	0.42	0.06
	2	*0.01*	0.08	*0.05*
	3	k	0.1	*0.02*

(i) Find the value of k.

(ii) Calculate the following probabilities.

(I) $Pr(X<1, Y=2)$ (II) $Pr(Y=3)$ (iii) $Pr(X<1)$

(iii) Find the marginal distribution of Y.

(iv) A student says that "the random variables X and Y are independent". State whether the students' statement is true or false. Justify your answer.

(v) Calculate $Pr(X > 0|Y = 3)$.

(vi) Calculate $E(X|Y = 3)$.

6.

Life time of a light bulb manufactured by ABC Company is normally distributed with mean 7 years and a standard deviation of 2 years.

(i) Find the probability that life time of a randomly selected light bulb exceeds two years.

(ii) The production manager of the ABC Company plans to set a warranty period (in years) such that 99% of the bulbs should not fail during the warranty period. Calculate the warranty period.

(iii) Random sample of 10 bulbs from the above population were tested and sample mean $\bar{X}$ was estimated. Find the probability that the sample mean $\bar{X}$ exceeds 7.5 years.

(iv) From past experience it is known that mean salary of employees in ABC company is Rs. 60000 with standard deviation of Rs. 2000. Suppose a sample of 100 employees were selected for a survey. Find the probability that the mean salary of the selected sample of employees will be less than Rs.60400? Clearly state any assumptions or theorems used in answering to this part

Left tail values of Standard Gamma Table
W - gamma(α,1)
This table contain the probabilities $Pr(W \leq w)$

	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432