The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme-2012/2013
Final Examination
Applied Mathematics-Level 05
AMU3183/AME5183 – Numerical Methods – II

Duration: Two Hours

Date: 31.05.2013 Time: 1.30pm-3.30pm

Answer Four Questions Only.

01. (a) Write down the n[th] order Lagrange's interpolation polynomial for the data $(x_0, y_0), (x_1, y_1), \ldots\ldots\ldots, (x_n, y_n)$.

(b) With the usual notation, prove that the error of Lagrange's interpolation method is $\frac{\pi(x)}{n+1!} f_{(c)}^{n+1}$, where $\pi(x) = (x - x_0)(x - x_1)\ldots\ldots\ldots(x - x_n)$ and $c \in (x_0, x_n)$.

(c) Some values of the function $f(x) = \ln(x+1)$ are tabulated as follows.

x	0	0.3	0.6
$y = \ln(x+1)$	0	0.26236	0.47000

(i) Construct the Lagrange's interpolating polynomial for the function. Hence find $f(0.45)$.

(ii) Using error bound, find the best approximation for the value $f(0.45)$.

02. (a) Starting from $y_1 = Ey_0$ construct the Newton's forward difference formula.

(b) The following data are part of a table for $g(x) = \frac{\sin x}{x^2}$

x	0.1	0.2	0.3	0.4	0.5
$g(x)$	9.9833	4.9667	3.2836	2.4339	1.9177

(i) Construct the Newton's interpolation formula for the function. Hence find $g(0.25)$.

(ii) Using error bound, find the best approximation for the value $g(0.25)$.

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03. (a) With the usual notation derive the least squares formulas to fit a straight line for a given set of data for fitting a straight line to a given set of data.

(b) Fit a curve of the type $y=\alpha e^{-\beta x}$ for the data given in table where α, β are real constant.

x	0.25	0.50	0.75	1.00	1.25	1.50
y	3.1	1.7	1.0	0.68	0.42	0.26

04. (a) Using Taylor Series expansion or otherwise derive the Euler Method to solve the first order initial value problem.

(b) Solve the differential equation

$$\frac{dy}{dx}+xy=0; \quad y(0)=1,$$

from $x=0$ to $x=0.25$ using Euler's method where $h=0.05$ for the appropriate decimal places.

05. (a) Using the second order Runge-Kutta method solve

$$\frac{dy}{dx}=1+y^2+x^3; \quad y(1)=-4, \ h=0.01.$$

Hence find $y(1.02)$.

(b) Using the fourth order Runge-Kutta method solve

$$\frac{dy}{dx}=1+y+x^2; \quad y(0)=0.5, \ h=0.2,$$

Hence find $y(0.2)$ and $y(0.4)$.

06. Write down the general formulas of the Modified Euler Predictor-Corrector method. Hence obtain $y(0.2)$ and $y(0.4)$ to the initial value problem

$$\frac{dy}{dt}=-\tan(y); \quad y(0)=1, \ h=0.2$$

with one correction at each time step.

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