The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 05
Final Examination 2012/2013
Applied Mathematics
APU 3145/ APE5145 – Newtonian Mechanics II

Duration :- Two Hours

Date :-09.12.2013 Time:-01.30 p.m. 03.30 p.m.

Answer Four Questions Only.

1. (a) State D' Alembert's principle.

(b) Two uniform spheres, each of mass M and radius a, are firmly fixed to the ends of two uniform thin rods of mass m and length l, and the other ends of the rods are freely hinged to a fixed point O. the whole system revolves about vertical axis through O with constant angular speed ω. Show that, when the motion is steady the rods are inclined to the vertical at an angle θ given by $\cos\theta = \dfrac{g}{\omega^2}.\dfrac{M(l+a)+\dfrac{ml}{2}}{M(l+a)^2+\dfrac{ml^2}{3}}$.

2. (a) In the usual notation, show that in spherical polar coordinates, the components of the velocity and acceleration of a particle are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\underline{k}$ and

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$$

respectively.

(b) A particle of mass m moves inside a smooth sphere. The velocity of the particle at a point P is the same as that due to it falling freely from rest from the level of the centre to the point P. Show that the reaction of the surface will vary as the depth below the centre.

3. (a) Obtain, in the usual notation, the equation $\dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega}\times\dfrac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) A projectile located at a point of latitude λ is projected with speed v_0 in a Northward direction at an angle α to the horizontal. Write down the equations necessary to determine the position $\underline{r}(t)$ of the particle at time t and solve for $\underline{r}(t)$.

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4. (a) With the usual notation, show that the Lagrange's equations of motion for a holonomic system are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0,\ \ j=1,2\ldots,n.$

(b) A uniform rod AB of length 2a is suspended from a fixed point O by a string OC of length $5a/6$ attached to a point C of the rod such that $AC=2a/3$. Choosing suitable coordinates $q_1,\ q_2$, show that the kinetic energy T is given by

$$2T=\frac{1}{36}ma^2\left(25\dot{q}_1^2+16\dot{q}_2^2+20\dot{q}_1\dot{q}_2\right) \text{ and } V=-mga\left(\frac{5}{6}\cos q_1+\frac{1}{3}\cos q_2\right)+Const.$$

Also, find the Lagrangian of the system and hence obtain the equations of motion.

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body then show that H and T are conserved.

(c) A solid cube is in motion about an angular point A which is fixed. If there are no external forces and $\omega_1, \omega_2, \omega_3$ are the angular velocities about the edges through A, prove that $\omega_1+\omega_2+\omega_3=$ constant and $\omega_1{}^2+\omega_2{}^2+\omega_3{}^2=$ constant.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i}=\dot{q}_i,\ \ \frac{\partial H}{\partial q_i}=-\dot{p}_i.$

(b) The Hamilton's of a dynamical system is given by

$$H=q_1p_1-q_2p_2-aq_1^2+bq_2^2 \qquad \text{where } a,\ b \text{ are constants.}$$

Obtain Hamilton's equations of motion and hence find q_1, q_2, p_1 and p_2 at time t.