The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2011/2012
AMU3187/ AME 5187- Mathematical Methods II
APPLIED MATHEMATICS-LEVEL 05

Duration: Two Hours.

Date: 20.01.2012	**Time: 1.30 p.m.- 3.30p.m.**

Answer FOUR questions only.

1. Consider the periodic function $f(x)$ defined by

$$f(x)=\begin{cases}-\cos x & \text{for } -\pi\le x<0\\ \cos x & \text{for } \quad 0<x\le\pi\end{cases}$$

and $f(x+2\pi)=f(x)$.

(i) Sketch the graph of $f(x)$ for two periods.

(ii) Find the Fourier series of $f(x)$.

(iii) Using part (ii) show that $\dfrac{\pi\sqrt{2}}{16}=\dfrac{1}{1.3}-\dfrac{3}{5.7}+\dfrac{5}{9.11}-\cdots$.

2. Consider the boundary value problem:

$$\frac{d^2y}{dx^2}+\mu y=0$$

$$y'(0)=0,\ y'(1)=0$$

(i) Show that this is a Sturm-Liouville problem.

(ii) Find the eigenvalues and eigenfunctions of the problem.

(iii) Verify that the eigenfunctions are mutually orthogonal in the interval $0\le x\le 1$.

(iv) Obtain a corresponding set of orthonormal functions in the interval $0\le x\le 1$.

3. Consider the function $f(x)$ defined by

$$f(x)=2x,\quad 0\le x\le\pi$$

(i) Find the Fourier sine series and cosine series of $f(x)$ on $0\le x\le\pi$.

(ii) Sketch the graphs of $f(x)$ for the two series.

(4) Let $J_n(x)$ be the Bessel function of order n given by the expansion

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n .$$

(i) Show that $J_n(x)$ is an even function when n is even and an odd function when n is odd.

(ii) Verify each of the following identities for n=1, 2, 3,....

(a) $J_{-n}(x) = (-1)^n J_n(x)$

(b) $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$.

(c) $\frac{d}{dx}\{x^{-n} J_n(x)\} = -x^{-n} J_{n+1}(x)$.

5 . The Rodrigues' formula for the n^{th} degree Legendre polynomial is given by

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n, \quad n=0, 1, 2, \ldots.$$

Using it prove the following identities:

(i) $P_{n+1}'(x) = (n+1)P_n(x) + xP_n'(x)$

(ii) $(\text{n}+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$

(iii) $(x^2-1)P_n'(x) = nxP_n(x) - nP_{n-1}(x)$.

(iv) $P_n(-x) = (-1)^n P_n(x)$ and $P_n'(-x) = (-1)^{n+1} P_n'(x)$

(6) Solve the following boundary value problem.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b.$$

$$\begin{aligned} u(x,0) &= f_1(x), && 0 < x < a \\ u(x,b) &= f_2(x), && 0 < x < a \\ u(0,y) &= g_1(y), && 0 < y < b \\ u(a,y) &= g_2(y), && 0 < y < b \end{aligned}$$