The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 05
Final Examination 2011/2012
Applied Mathematics
APU 3145/ APE5145 – Newtonian Mechanics II

Duration :- Two Hours

Date :-26.11.2012 **Time:-01.30 p.m. 03.30 p.m.**

Answer Four Questions Only.

1. (a) State D' Alembert's principle.

   (b) One end of an inextensible string of length l is attached to a fixed point O on a ceiling and the other end to the end A of a uniform rod AB of length $2a$, the rod being suspended by the string. If both the rod and the string revolve about the vertical with uniform angular velocity and if their inclinations to the vertical are θ and ϕ respectively, show that $\frac{3l}{a}=\frac{(4\tan\phi-3\tan\theta)\sin\theta}{(\tan\theta-\tan\phi)\sin\phi}$.

2. (a) Show that in spherical polar coordinates, the components of the velocity and acceleration of a particle are given by $\underline{\dot{r}}=\dot{r}\hat{r}+r\dot{\theta}\hat{\theta}+r\sin\theta\dot{\phi}\hat{\phi}$ and
$\underline{\ddot{r}}=\left(\ddot{r}-r\dot{\theta}^2-r\dot{\phi}^2\sin^2\theta\right)\hat{r}+\left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)-r\sin\theta\cos\theta\dot{\phi}^2\right)\hat{\theta}+\frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$
respectively.

   (b) A particle is projected horizontally with velocity u along the interior surface of a smooth hemisphere whose axis is vertical and whose vertex is downwards. The radius through the point of projection makes angle β with the downward vertical. If the particle just ascends to the point of projection show that $u=\sqrt{2ag\sec\beta}$, where a is the radius of the hemisphere.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2\underline{r}}{\partial t^2}+2\underline{\omega}\times\frac{\partial\underline{r}}{\partial t}=-g\underline{k}$ for the motion of a particle relative to the rotating earth.

   (b) An object is projected vertically downward with speed v_0 from a point O, near the surface of the earth, having latitude λ. Prove that after time t, the object is deflected east of the vertical by an amount $\omega v_0\cos\lambda t^2+\frac{1}{3}\omega g t^3\cos\lambda$, where ω is the angular speed of earth about its polar axis.