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THE OPEN UNIVERSITY OF SRI LANKA
Department of Civil Engineering
Bachelor of Technology - Level 5

CEX 5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION - 2008/2009

Time Allowed : **Three Hours** *Index No.* [ ]

Date : 28th March, 2009 Time : 1330 - 1630

ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

PART A

Answer **all three** questions in this section.

1) A long, straight, horizontal open channel has a uniform, rectangular cross-section with a width of **b** . The channel has a section AB over which a series of small holes have been made in one wall of the channel, as shown in Figure 1a. The holes have a diameter **d** and their centres are at a spacing of **c** as shown. When water flows in the channel between A and B, some water is discharged from these holes to the atmosphere, as shown in Figure 1b.

a) Obtain an expression for the discharge from a single hole in terms of **E(x)** , where **E(x)** is the specific energy of the flow in the channel at that point. Assume that the flow depth **y(x)** is much greater than the diameter of the holes **d** .

b) Apply the principle of conservation of mass to the control volume of length **δx** shown in Figure 1a and obtain the equation $\frac{dQ(x)}{dx} = -\frac{C_d \pi d^2}{4c}\sqrt{2gE(x)}$ where **Q(x)** is the channel discharge at a given point.

c) Assuming that there are no energy losses between A and B, differentiate the expression for the specific energy in the channel to obtain the equation $\frac{dy}{dx} = -\frac{Qy\left(\frac{dQ}{dx}\right)}{gb^2y^3 - Q^2}$

d) Use the equation derived in section c) to show that the flow depth increases between A and B if the flow is sub-critical at A.

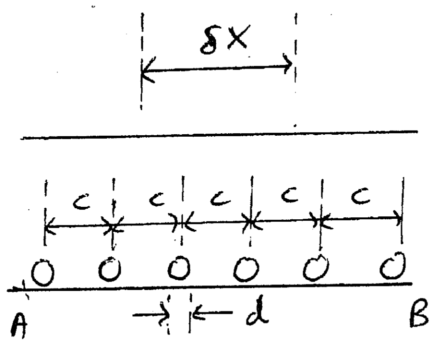

FIGURE 1a

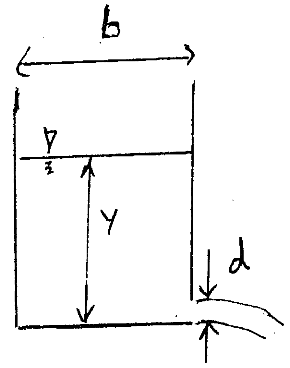

FIGURE 1a

2) The power, $\mathbf{P}$, generated by a wind turbine of a certain geometry, depends on the diameter of the turbine, $\mathbf{d}$, the velocity of the wind, $\mathbf{V}$, the rate of rotation of the turbine, $\mathbf{N}$, (in revolutions per unit time), the density of the air, ρ, and the viscosity of the air, μ.

A wind turbine has a diameter of 15 m and is to be designed to operate in a wind of 10 m/s. The design is to be tested by conducting a model study at a scale of 1:20. The model study is to be conducted using air as the working fluid. The density of air is 1.25 kg/m^3 and the dynamic viscosity of air is 1.6×10^{-5} Pa s.

a) Derive a relationship between the non-dimensional power generated by the turbine and other relevant non-dimensional quantities.

b) When the model test is conducted using a constant wind speed show in a neat sketch how the power generated by the turbine would vary with the rate of rotation of the turbine. Explain why the power generated varies in this way.

c) An air velocity of 10 m/s is selected for the model study. Consider the dynamic similarity between model and prototype and discuss whether this is a suitable velocity for the model study.

The study shows that a maximum power of 350 W can be extracted from the model turbine when the speed of rotation is 200 revolutions per minute.

d) Calculate the operating rate of rotation and the maximum power that can be extracted for the prototype turbine.

3) A long open channel ABC has a rectangular cross-section with a width of 2 m, a slope of 0.002 and a Manning's coefficient of 0.015. There is a sluice gate B that has an opening of 0.2 m. The channel ends in a free overfall at C, as shown in Figure 3. It is observed that there is a hydraulic jump between B and C.

a) If the depth immediately upstream of the sluice gate is 1 m, calculate the discharge in the channel. Assume that there are no energy losses and flow contraction as the flow passes under the gate.

b) Does the channel have a mild slope or a steep slope for this discharge?

c) Sketch one possible free surface profile between B and C and classify the elements of the profile (from M1, M2, M3, S1, S2, S3, C1, C2, C3, etc.). Explain your answer.

d) Explain how you would estimate the location of the hydraulic jump between B and C.

e) For the free surface profile sketched in d), explain how the location of the jump would change if the sluice gate at B was raised slightly. Assume that raising the sluice gate does not change the discharge in the channel.

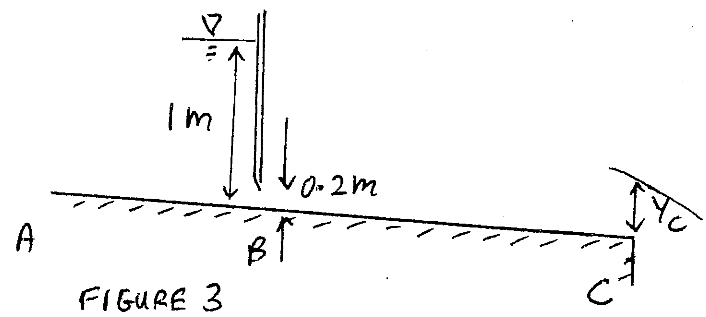

FIGURE 3

PART B

Answer any **two** of the **five** questions in this section.

4) The velocity vector, **U** , in a two-dimensional flow can be written as $U = u\,\underline{i} + v\,\underline{j}$ where **i** and **j** are the unit vectors in the **x** and **y** directions and **u** and **v** are the velocity components in the x and y directions. The stream function ψ and the potential function ϕ are defined by the equations $u = \dfrac{\partial \psi}{\partial y}$ and $v = -\dfrac{\partial \psi}{\partial x}$ and $u = \dfrac{\partial \phi}{\partial x}$ and $v = \dfrac{\partial \phi}{\partial y}$, respectively.

a) What is the condition satisfied by the velocity vector for irrotational flow?

b) Show that the stream function satisfies the Laplace equation (i.e. $\nabla^2 \psi = 0$) when the flow is irrotational.

c) What is the simplified form of the equation for mass conservation in a steady, incompressible, two-dimensional flow?

d) Show that the potential function satisfies the Laplace equation (i.e. $\nabla^2 \phi = 0$) when the flow is steady and incompressible.

e) What is the two-dimensional ideal fluid flow corresponding to the function $w = z^5$? Explain your answer.

f) Sketch the streamlines of the flow defined in section e).

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5) The Meyer-Peter-Muller formula for bed load transport is given by

$$\frac{q_s \rho^{1/2}}{(\rho(s-1)gd)^{3/2}} = 8\left(\frac{\rho u_*^2}{\rho(s-1)gd} - 0.047\right)$$

where $\mathbf{q_s}$ is the rate of sediment transport, ρ the density of water, **s** the specific gravity of the sediment, **d** the grain diameter and $\mathbf{u_*}$ the shear velocity ($u_* = \sqrt{\dfrac{\tau_0}{\rho}}$).

a) Explain what is meant by the "initiation of motion" when considering sediment transport in rivers.

b) Discuss the relationship between the concept of "initiation of motion" and the Meyer-Peter-Muller formula for bed load transport given above.

c) What are the units of the rate of sediment transport $\mathbf{q_s}$ in this equation?

A long straight river is 20 m wide and has a slope of 0.0001. The river bottom consists of fine sand of diameter 0.1 mm and specific gravity 2.65 . The flow depth in the river is 3 m .

d) Calculate the discharge in the river if the flow is uniform. Assume that Manning's n for a flat sand bed is given by $n = 0.039 d^{1/6}$ where d is the grain diameter in metres.

e) In a real river with a sand bed would the value of Manning's n be greater than or less than the value you used in section d)? Explain your answer.

f) Calculate the rate of sediment transport in the river.

6) A long, straight river flows in a long, straight area of deposited sediment that is bounded below and on both sides by impervious rock as shown in Figure 6. The deposited sediment is homogenous and isotropic and has a permeability of 2.5 m/day . The area is recharged at an average rate of **q** mm/day due to the infiltration of rainfall.

A long straight irrigation channel flows parallel to the river at a distance of 1500 m from the river, as shown in the figure. The water level in the canal is 10 m above the water level in the river. As the canal is not lined, water can seep from the canal to the ground or from the ground to the canal depending on whether the water level in the canal is below or above the groundwater level near the canal.

a) Derive, from first principles, an equation governing the variation of the groundwater level between the canal and the river. State all your assumptions.

b) What are the boundary conditions needed to solve this equation? Explain your answer.

c) Calculate the recharge rate, **q** , for which there will be no exchange of water between the canal and the ground. Explain your answer.

d) Explain what will happen if the recharge rate is greater than the value calculated in section c).

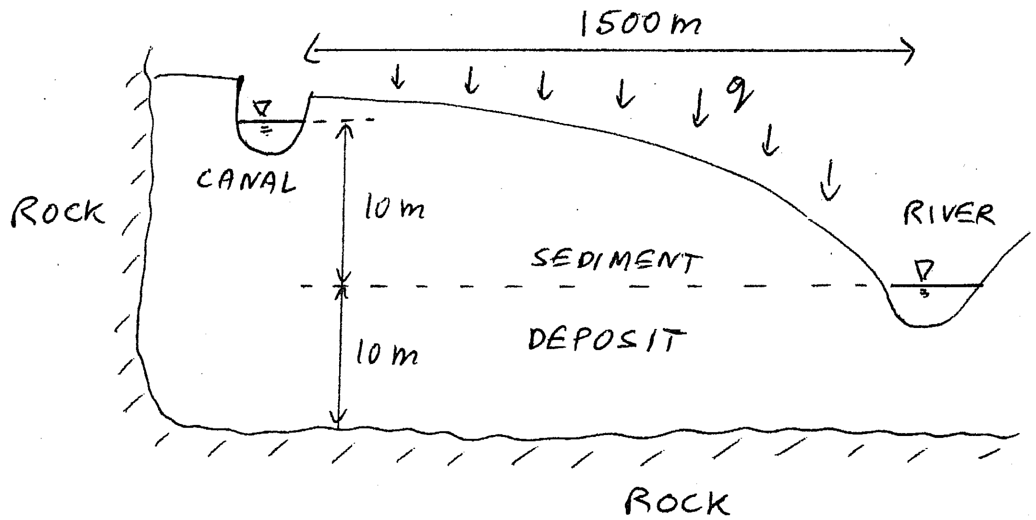

FIGURE 6

7) A pump, P_1, is used to pump water from one reservoir to another as shown in Figure 7. The difference in elevation between the surfaces of the reservoirs is 20 m. The pipeline has a diameter of 150 mm, a friction factor of 0.012 and a length of 2 km. The performance characteristics of the pump at its operating speed are given in Table 7a.

a) Calculate the discharge in the pipeline when only P_1 is operated. Explain your answer.

A second pump, P_2, is connected in parallel with P_1 in order to increase the discharge. The performance characteristics of P_2 are given in Table 7b.

b) Calculate the discharge in the pipeline when both pumps, P_1 and P_2, are in operation. Explain your answer.

Head (m)	40	35	30	20	10
Discharge (l/s)	0	10	20	30	40

Table 7a : Performance Characteristics of Pump P_1

Head (m)	40	35	30	20	10
Discharge (l/s)	0	5	10	15	20

Table 7b : Performance Characteristics of Pump P_2

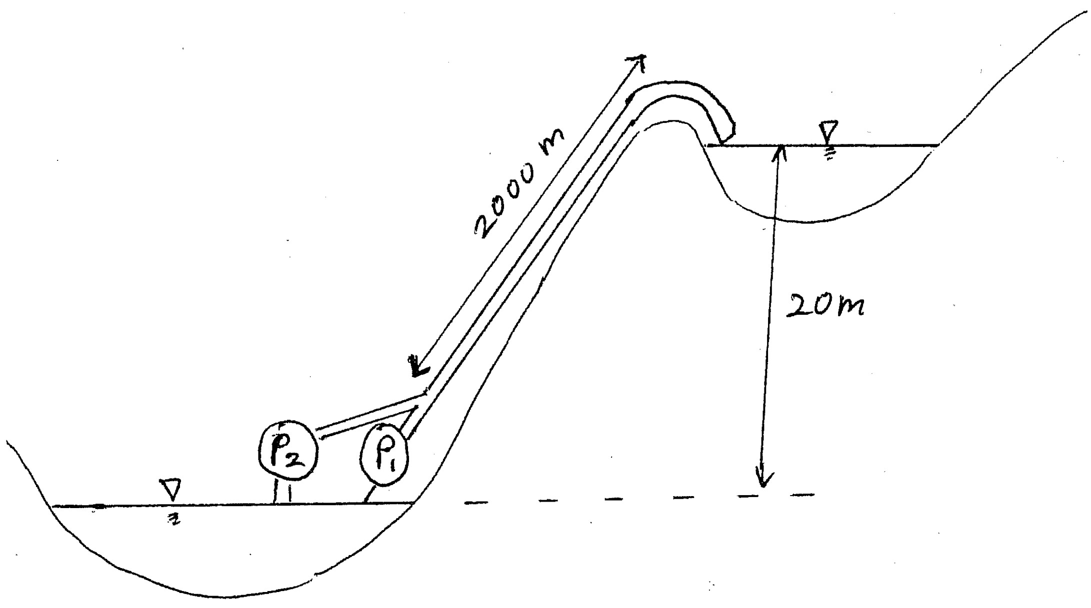

FIGURE 7

8) A kite is made by pasting paper over a framework of ekels. The shape of the kite is a curved rectangle. A side view of the kite, when it is flying stably, is shown in Figure 8. The wind is blowing from right to left as shown.

a) Explain what is meant by the Drag Force and the Lift Force on a body held stationary in a flowing fluid.

b) Explain what is meant by flow separation.

c) Using a sketch of the streamlines of the wind flow past the kite, explain how flow separation can result in Drag and Lift Forces on the kite.

d) Mark all the forces acting on the kite on a neat diagram.

e) Write down the equations representing the balance of these forces when the kite is flying in a stable position.

f) Explain why pulling on the string will cause a kite to rise when it is flying in a uniform wind.

g) Is there a maximum height to which a kite will fly if the wind is uniform (i.e. the velocity of the wind is constant with height from the ground) and the person flying the kite is not moving? Explain your answer.

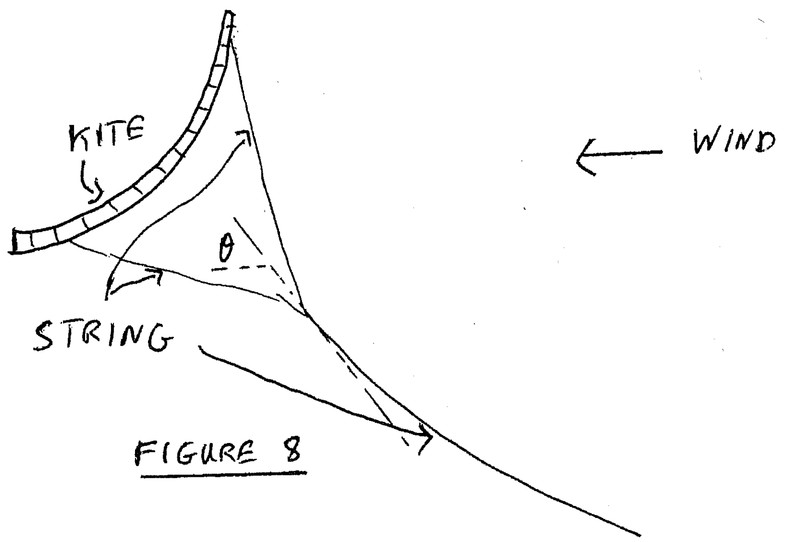

FIGURE 8