The Open University of Sri Lanka Department of Mathematics B.Sc/ B.Ed Degree Programme No Book Test (NBT) - 2017/2018 Applied Mathematics— Level 05 ADU5308 — Graph Theory



**DURATION: ONE HOUR** 

Date: 10 - 02 - 2019 Time: 4.00 p.m. - 5.00 p.m.

## ANSWER ALL QUESTIONS. THE TOTAL MAXIMUM MARK ATTAINABLE IS 200 AND THE FINAL MARK WILL BE CONVERTED TO 100%.

01. Draw the digraph  $D_1 = (V_1, A_1)$  whose adjacency list is given in the following table.

[15 Marks]

Vertex	Adjacent Vertices	Vertex	Adjacent Vertices	Vertex	Adjacent Vertices
t	W	v	t,z	X	y,w
u	x,t	w	z, v	у	v, x, z

- (a) Find the minimum length of each of the vertices
  - (i) from the vertex u,

[15 Marks]

(ii) to the vertex z, among all the walks in the digraph  $D_1$ .

[15 Marks]

Hence, determine whether  $D_1$  is *strong* or not.

[10 Marks]

(b) Let  $D_2 = (V_1, A_2)$ , where  $A_2 = A_1 \cup \{(z, u)\}$ . Show that  $D_2$  is strong.

[15 Marks]

Is  $D_2$  a tournament? Justify your answer.

[10 Marks]

- 02. Let  $L(K_{1,3} + x)$  be the *line graph* of the graph  $K_{1,3} + x$ , where x is a *line incident* with two of the *points* having *degree* one in  $K_{1,3}$ .
  - (a) Without drawing the graph  $L(K_{1,3} + x)$ ,
    - (i) determine the number of lines in  $L(K_{1,3} + x)$ ,
    - (ii) find the *degree* of each of the *points* in  $L(K_{1,3} + x)$ .

[50 Marks]

(b) Draw  $L(K_{1,3} + x)$  and  $L^2(K_{1,3} + x)$ . Hence, verify the results obtained in part (a).

[40 Marks]

(c) Show that  $L(K_{1,3} + x)$  and  $K_4 - y$  are isomorphic, where y is any line deleted from  $K_4$ .

[20 Marks]

Hence, deduce that  $K_4 - y$  is a line graph.

[10 Marks]

