The Open University of Sri Lanka B.Sc/B.Ed. Degree Programme Open Book Test (OBT) - 2017/2018 Pure Mathematics - Level 05 PEU5300- Riemann Integration



Open Book Test Duration: 1 hour

Calculators not allowed

Date: 05th January 2019

Time: 1.00p.m -2.00p.m

Answer all questions

Q1)

- (i) Let $f(x) = 3\cos x + 7$ and P be the partition of $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$ having four sub-intervals of equal width. Find U(P, f) and L(P, f).
- (ii) Let $g: [1,3] \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} 0, & x \le 2\\ 0, & x \in (2,3] \cap \mathbb{Q}\\ 1, & x \in (2,3] \cap \mathbb{Q}^c \end{cases}$$

Prove that g is not Riemann integrable on [1, 3].

(Q2)

Let $h: [0,3] \to \mathbb{R}$ be defined by

$$h(x) = \begin{cases} 1, & 0 \le x < 1 \\ 2, & x = 1 \\ 3, & 1 < x \le 2 \\ 4, & 2 < x \le 3 \end{cases}$$

- (i) Find the lower and upper integrals of h on [0,3].
- (ii) Prove that h is Riemann integrable on [0,3] and $\int_0^3 h(x)dx = 8$.

