THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY/ BACHELOR OF SOFTWARE ENGINEERING – LEVEL 05
FINAL EXAMINATION – 2010/2011
MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II
DURATION : THREE (03) HOURS.

Date : 02nd March 2011 **Time . 930-1230 hrs.**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define a group in usual notation.

ii. Show that $(R-\{0\},*)$ is a group, where $*$ denotes the usual multiplication of real numbers.

iii. Let $G=\{1,-1\}$. Show that (G, $*$) is a group where $*$ is the ordinary multiplication.

iv. Prove that Q is a group with respect to the binary operation $*$ defined by $a*b=a+b-2ab$

02. i. Show that the identity element in a group G is unique.

ii. Let A be any nonempty set with the operation $a*b=a$. Is the operation

a) associative?

b) commutative?

iii. Let G be the set of all 2 x 2 square matrixes $\begin{pmatrix} m & m \\ m & m \end{pmatrix}$ such that m is a positive real number. Show that G together with the matrix multiplication forms abelian group.

03. i. Define a Homomorphism for groups in usual notation.

ii. Let G be the group of real numbers under addition, and let G' be the group of positive real numbers under multiplication. Show that the mapping $f: G \to G'$, defined by $f(a) = 2^a$, is a homomorphism.

iii. If ϕ is a homomorphism of G into G', then show that

a) $\phi(e) = e'$, where e and e' are identities in G and G' respectively.

b) $\phi(x^{-1}) = [\phi(x)]^{-1}$; $\forall x \in G$

SECTION – B

04. i. Draw each of the following graphs and identify the connected and disconnected graph.

a) $G = \{V, E\}$ *Where* $V = \{a, b, c, d, e, f, g, h, i\}$
and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ with
$e_1 = \{a, b\}$, $e_2 = \{a, c\}$, $e_3 = \{b, d\}$, $e_4 = \{b, e\}$, $e_5 = \{d, h\}$
$e_6 = \{e, h\}$, $e_7 = \{c, f\}$, $e_8 = \{c, g\}$, $e_9 = \{f, i\}$, $e_{10} = \{g, i\}$

b) $G = \{V, E\}$, *where* $V = \{n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8\}$

and $E = \{(n_1, n_2), (n_2, n_7), (n_1, n_8), (n_3, n_4), (n_3, n_6), (n_4, n_5), (n_5, n_6), (n_7, n_8)\}$

ii. Draw the graph representing the following relationships.

Pasindu likes to work with Menuka, Amal and Kamal but dislikes to work with Nimal and Saman, Nimal and Saman like to work with each other, but both dislike to work with Pasindu, Menuka, Amal and Kamal. Menuka, Amal and Kamal like to work with each other.

iii. Briefly explain simple graph, multigraph and subgraph.

iv. Find the vertices n such that the complete graph, has at least 500 edges.

05. i. Show that in any connected graph, there are even number of odd degree vertices.

ii. G is the graph whose adjacency matrix A is given by

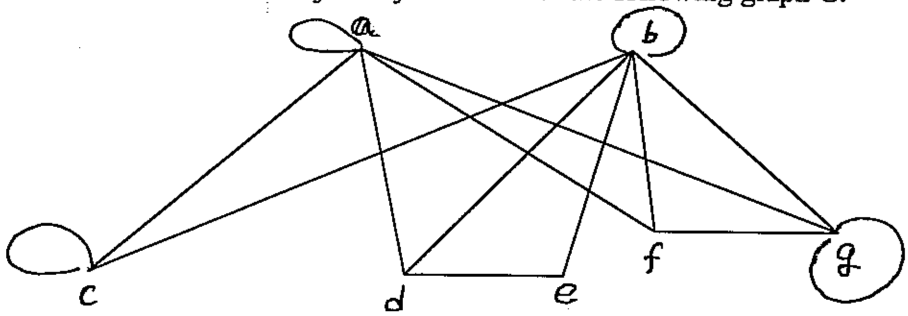
$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

a) If $V(G) = \{v_1, v_2, v_3, v_4\}$, find the number of paths of length four (4) joining vertices v_2 and v_4.

b) Without drawing a diagram of G, determine whether G is connected or not.

c) Draw the graph of adjacency matrix A.

iii. Find the adjacency matrix M of the following graph G.



06. i. Let T be a graph on n vertices. Show that if T is connected and cylic the T is connected and has n – 1 edges.

ii. Draw all trees with five vertices.

iii. Draw a tree with 10 nodes each of which has either degree 1 or degree 3. Show that it is impossible to draw such a tree with 11 nodes. Repeat the problem by showing that you can draw such a tree with 16 nodes but that it is impossible to do so for 15 nodes. Characterize the size of the node set for which such a tree exists.

SECTION – C

07. i. Draw the graph and show the graphical solutions of $x_{n+1} - \lambda x_n = 0$ for $x_0 = 0.4$, Taking $\lambda - 0.4$ and $\lambda = 1.45$.

ii. Explain mathematically as to why used the equation $x_{n+1} = \lambda x_n(1 - x_n)$ to represent the population growth in a limited eco-system, instead of the equation $x_{n+1} = \lambda x_n$.

iii. Iterate the relation and draw the diagram $Z_{n+1} = Z_n^2 + \lambda$ find $\lambda = 0$ and indicate the initial condition $Z_0 = 2.2 + i0.5$ and find Z_{10} and hence find Zn as $n \to \infty$.

08. A three dimensional system is governed by the following three differential equation.

$$\frac{dx}{dt} = 5x + 2y + 4z$$

$$\frac{dy}{dt} = -3x + 6y + 2z$$

$$\frac{dz}{dt} = 3x - 3y + z$$

at t = 0 , (x,y,z) = (1,1,0)

Find the phase space value (x_n, y_n, z_n) for $n = 1,2$.

09. i. Consider the language L = {ab,c} over A = {a,b,c} Find L^3.

ii. Define a finite state machine and Deterministic Finite Automation (DFA).

iii. Draw the directed graph that describes the DFA with the following state transition table.

States	Input	
	a	b
S_0	S_1	S_3
S_1	S_1	S_2
S_2	S_3	S_2
S_3	S_2	S_3

Initial state S_0 and accepting state S_2. Also construct an automation M which will accept the language $L = \{a^m b^m : m \text{ and } n \text{ positive}\}$.

iv. The two from binary pipe line devise hold up two binary digit as in following table.

Contents	Input		Output	
	0	1	0	1
00	00	10	0	1
01	01	11	1	0
10	01	11	0	0
11	10	00	1	1

Let M be a mealy machine. Let $s \in S$, $a \in I$ and $x \in I*$ and define function $\delta^* : S \times I^* \to S$ and $\beta^* : S \times I^* \to O^*$ by

$$\delta^*(S, \Omega) = S$$
$$\delta^*(S, a \,.\, x') = \delta^*[\delta(S, a), x']$$
$$\beta^*(S, \Omega) = \Omega$$
$$\beta^*(S, a.x') = \beta(s, a).\beta^*[\delta(s, a), x']$$

Find the two frame binary pipe line buffer and work out its response to the sequence 1010 from state 10.