The Open University of Sri Lanka

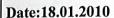
B.Sc./B.Ed. Degree Programme

Final Examination-2009/2010

AMU 3187/ AME 5187- Mathematical Methods II

APPLIED MATHEMATICS-LEVEL 05

Duration: Two and Half Hours.



Time: 1.00 p.m.- 3.30p.m.

Answer FOUR questions only.

- (1) (a) Define a periodic function.
 - (b) Consider the periodic function f defined by

$$f(x) = \begin{cases} 1 + \frac{x}{\pi} & \text{for } -\pi \le x \le 0 \\ 1 - \frac{x}{\pi} & \text{for } 0 \le x \le \pi \end{cases}$$

and
$$f(x+2\pi) = f(x)$$

- (i) Find the Fourier series of f.
- (ii) Using part (i), show that

$$\sum_{r=1}^{\infty} \frac{1}{(2r-1)^2} = \frac{\pi^2}{8}.$$

- (2) Consider the function f defined on 0 < t < 1 by $f(t) = t t^2$.
 - (a) Sketch both the even and odd periodic extensions of f for two periods each.
 - (b) Find the half-range expansions of f.
 - (c) Show that

(i)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3} = \frac{\pi^3}{32}$$
.

(ii)
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2} = \frac{\pi^2}{48}$$
.



- (3) (a) Let $f_1(x)$, $f_2(x)$, $f_3(x)$,... be a set of real-valued functions, which are orthogonal with respect to the weight function p(x) on the interval $a \le x \le b$. If $h_m(x) = \sqrt{p(x)} f_m(x)$; (m = 1, 2, 3,...), then show that $h_1(x)$, $h_2(x)$, $h_3(x)$... are orthogonal on the interval $a \le x \le b$.
 - (b) (i) Let $f_1(x) = a$, $f_2(x) = 1 bx$ and $f_3(x) = 1 cx + dx^2$ where a, b, c and d are non-zero constants. Suppose that $f_1(x)$, $f_2(x)$, $f_3(x)$ are orthogonal in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$. If $||f_1(x)|| = 1$, find the values of a, b, c and d.
 - (ii) Show that $\{f_1(x), f_2(x), f_3(x)\}$ forms an orthonormal set in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$.
- (4) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \mu y = 0$$

y(-2) = y(2)
y'(-2) = y'(2)

- (i) Show that this is a Sturm-Liouville problem.
- (ii) Find the eigenvalues and eigenfunctions of the problem.
- (iii) Obtain a set of functions which are orthonormal in the interval $-2 \le x \le 2$.
- (5) Let $J_n(x)$ be the Bessel function of order n given by the expansion

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n.$$

Verify each of the following identities for n=1, 2, 3, ...

(i)
$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$

(ii)
$$J'_{n}(x) = \frac{1}{2} [J_{n-1}(x) - J_{n+1}(x)]$$

(iii)
$$J'_{n}(x) + \frac{n}{x}J_{n}(x) = J_{n-1}(x)$$

(iv)
$$\frac{d}{dx}[J_n^2(x)] = \frac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)]$$

(6) Solve the following boundary value problem with mixed boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b.$$

$$\frac{\partial u}{\partial x}(0, y) = 0, \quad 0 < y < b.$$

$$\frac{\partial u}{\partial x}(a, y) = 0, \qquad 0 < y < b.$$

$$u(x, 0) = 0, \qquad 0 < x < a.$$

$$u(x, 0) = 0,$$
 $0 < x < a.$
 $u(x, b) = 1,$ $0 < x < a.$

