The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2009/2010
AMU 3187/ AME 5187- Mathematical Methods II
APPLIED MATHEMATICS-LEVEL 05

Duration: Two and Half Hours.

Date:18.01.2010	**Time: 1.00 p.m.- 3.30p.m.**

Answer FOUR questions only.

(1) (a) Define a periodic function.

(b) Consider the periodic function f defined by

$$f(x)=\begin{cases} 1+\dfrac{x}{\pi} & for\ \ -\pi\le x\le 0 \\ 1-\dfrac{x}{\pi} & for\ \ 0\le x\le \pi \end{cases}$$

and $f(x+2\pi)=f(x)$

(i) Find the Fourier series of f.

(ii) Using part (i), show that

$$\sum_{r=1}^{\infty}\frac{1}{(2r-1)^2}=\frac{\pi^2}{8}.$$

(2) Consider the function f defined on $0< t <1$ by $f(t)=t-t^2$.

(a) Sketch both the even and odd periodic extensions of f for two periods each.

(b) Find the half-range expansions of f.

(c) Show that

(i) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3}=\frac{\pi^3}{32}.$

(ii) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2}=\frac{\pi^2}{48}.$

(3) (a) Let $f_1(x),\ f_2(x),\ f_3(x),...$ be a set of real-valued functions, which are orthogonal with respect to the weight function $p(x)$ on the interval $a \le x \le b$. If $h_m(x) = \sqrt{p(x)}\, f_m(x)$; ($m = 1, 2, 3,...$), then show that $h_1(x)$, $h_2(x), h_3(x)$ ...are orthogonal on the interval $a \le x \le b$.

(b) (i) Let $f_1(x) = a$, $f_2(x) = 1 - bx$ and $f_3(x) = 1 - cx + dx^2$ where a, b, c and d are non-zero constants. Suppose that $f_1(x), f_2(x), f_3(x)$ are orthogonal in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$. If $\|f_1(x)\| = 1$, find the values of a, b, c and d.

(ii) Show that $\{f_1(x), f_2(x), f_3(x)\}$ forms an orthonormal set in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$.

(4) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \mu y = 0$$
$$y(-2) = y(2)$$
$$y'(-2) = y'(2)$$

(i) Show that this is a Sturm-Liouville problem.
(ii) Find the eigenvalues and eigenfunctions of the problem.
(iii) Obtain a set of functions which are orthonormal in the interval $-2 \le x \le 2$.

(5) Let $J_n(x)$ be the Bessel function of order n given by the expansion

$$e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n .$$

Verify each of the following identities for n=1, 2, 3,....

(i) $J_{n+1}(x) = \dfrac{2n}{x} J_n(x) - J_{n-1}(x)$

(ii) $J'_n(x) = \dfrac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$

(iii) $J'_n(x) + \dfrac{n}{x} J_n(x) = J_{n-1}(x)$

(iv) $\dfrac{d}{dx}[J_n^2(x)] = \dfrac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)]$

(6) Solve the following boundary value problem with mixed boundary conditions.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0<x<a, \quad 0<y<b.$$

$$\frac{\partial u}{\partial x}(0, y)=0, \qquad 0<y<b.$$

$$\frac{\partial u}{\partial x}(a, y)=0, \qquad 0<y<b.$$

$$u(x, 0)=0, \qquad 0<x<a.$$

$$u(x, b)=1, \qquad 0<x<a.$$