THE OPEN UNIVERSITY OF SRI LANKA - CITES
Department of Mathematics and Philosophy of Engineering
Bachelor of Software Engineering Honours Study Programme

Final Examination 2023/2024

MHZ3459: Basic Mathematics for Computing

Date: 16th January 2025 **Time: 09:30 hrs. - 12:30 hrs.**

Instructions:

- This paper consists of Five (5) questions in Two (2) pages.
- Answer to ONLY Four(4) questions.
- Show all your workings.

1. (a) Are the points $A = (1, 2)$, $B = (-1, 3)$, $C = (4, 1)$ co-linear? Justify your answer. If points are not co-linear find the area of triangle ABC. [30%]

   (b) Let $A = \begin{bmatrix} 3 & -3 & -3 \\ -7 & 1 & -4 \\ 0 & -2 & 3 \end{bmatrix}$.

   i. If $A^3 - 7A^2 - 14A + 120I_3 = 0_3$ find A^{-1}, where I_3 and 0_3 is the 3×3 identity matrix and the zero matrix respectively. [40%]

   ii. Solve the following system of linear equations by using the method of matrices inverse.

$$\begin{aligned} 3x - 3y - 3z &= 3 \\ -7x + y - 4z &= -2 \\ -2y + 3z &= 1. \end{aligned}$$

[30%]

2. (a) Define $f(x) = \left(\ln\left(\frac{x}{2}\right) - e^{-3x}\right)^3$ for $x > 0$. Find the derivative of f. [40%]

   (b) A balloon is inflating in such a way that the volume is increasing at a rate of $r^3 - 4r^2 + 5r$ cubic centimeters per second, where r is the radius of the balloon in centimeters. Assume that the balloon is spherical. Determine how quickly the balloon's surface area changes when the radius is 2 centimeters. [60%]

3. (a) Consider the sequence $100, 97, 94, \cdots$. Determine how many terms must be added in order for the total terms to equal zero, if at all possible. [30%]

   (b) Evaluate $\lim_{x \to 0} \frac{\cos(9x) - \cos(3x)}{\cos(8x) - \cos(2x)}$. [30%]

   (c) Evaluate $\int \ln(\sin x)\cos(2x)dx$ using the method of integration by parts. [40%]

4. (a) The time taken to compile a software project of size 10MB, 15MB, and 20MB are 30 seconds, 45 seconds, and 70 seconds respectively. Use Newton's backward difference interpolation to approximate the time taken to compile a 17MB software project to the nearest second. [50%]

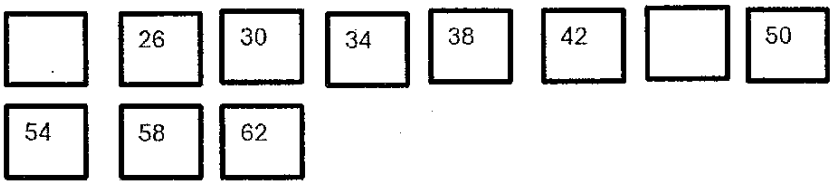
   (b) Let $f(x) = x^3 - 2x - 5$ on $\mathbb{R}$.

      i. Show that the Newton-Raphson method leads to the recurrence

$$x_{n+1} = \frac{2x_n^3 + 5}{3x_n^2 - 2} \text{ for } n \in \mathbb{N}. \qquad [10\%]$$

      ii. Starting with the initial value $x_0 = 2$ find the solution to $f(x) = 0$ to the nearest 3 decimal places. [40%]

5. (a) The time (in minutes) taken by 11 distinct servers to respond to a high-load stress test is listed below, in ascending order. However, two values are not available.



      i. If the range of the response times of the servers is 45 minutes, find the smallest missing value. [20%]

      ii. If the mode of the time is 42, find the second missing value. [20%]

      iii. Using the answers in parts (i) and (ii), find the Median and Mean of this data and discuss the shape of the distribution. [20%]

   (b) A software development company is creating two features (X and Y) for a new application. The two development teams, Team A and Team B, work on these elements independently. Both teams intend to have their features ready for the app's launch. Team A has a 70% chance of finishing feature X on time, whereas Team B has a 60% chance of completing feature Y on time.

      i. Compute the probability that neither team completed their feature on time. [20%]

      ii. Obtain the probability that at least one feature will be completed on time. [20%]

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