THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

AA1

Final Examination (2023/2024)

MHZ4256: Mathematics for Computing

Date: 21st August 2024 (Wednesday) **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- *Answer six (06) questions by selecting at least **one question from each of sections A, B and C.***
- *This paper consists of **nine (09)** questions.*
- *All the symbols are in standard notation unless they are defined.*

Section – A

Q1. **a).** State whether each of the following statements is a proposition or not. Explain the truth value of each. 15%

i). $\forall x \in \mathbb{R}^{+}(x < -4 \Rightarrow x^3 = 1000)$.

ii). $\forall x \in \mathbb{R}(x = -y \Rightarrow x^2 y = 1000)$.

iii). The function $f(x) = cosx$, where $x \in \mathbb{R}$, is a periodic function and π is an even number.

b). Find the converse, inverse and contrapositive propositions of the following proposition. 15%

If the formula is correct, then the solutions are correct.

c). Let p and q be propositions. 55%

i). Construct one truth table for the propositions $\neg p$, $p \vee q$, $(p \Rightarrow q)$, $p \wedge q$, $\neg p \vee q$ and $p \Leftrightarrow q$.

ii). Show that $p \Leftrightarrow q$ is a logical consequence of $p \Rightarrow q$, $p \wedge q$ and $p \vee q$.

iii). Show that $\big((p \Rightarrow q) \wedge (p \vee q)\big) \Rightarrow (p \Leftrightarrow q)$ is a tautology.

iv). What is the relation between $p \Rightarrow q$ and $\neg p \vee q$?

d). Let p, q and r be propositions. By using laws of the algebra of propositions, prove that $(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \equiv p \vee q$. 15%

Q2. **a).** Determine the truth values of the following statement and write down the negation of the following statement. 30%

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \big((|x| > |y|) \Rightarrow (x^2 > y^2)\big)$$

b). Find the truth value of $(\forall x)\Big(Q(x) \Rightarrow P\big(a,\ g(x)\big)\Big)$. Given that 30%

$D = \{0,\ 1\},\ a = 1,\ g(0) = 1,\ g(1) = 0,\ Q(0) = F,\ Q(1) = T$

$P(0,\ 0) = T,\ \ P(0,\ 1) = T,\ \ P(1,\ 0) = F,\ \ P(1,\ 1) = T.$

c). Using the method of direct proof, prove that for all $n \in \mathbb{Z}$, if $7n + 9$ is even, then n is odd. 20%

d). By using the method of proof by contrapositive, prove that if $2|n^3$, then $2|n$. 20%

Q3. **a).** Let $x,\ y, \in \mathbb{R}$. Using the method of proof by contradiction, prove that 30%

$$\forall x \forall y \left(6x + 5y > 5 \Rightarrow \left(x > \frac{8}{7} \vee y > \frac{7}{15}\right)\right).$$

b). By using the principle of mathematical induction, prove that $8^{n+1} + 9^{2n-1}$ is divisible by 73. 30%

c). Using the proof by case, show that $2n^2 + 5n + 4$ is not divisible by 3, when $n \in \mathbb{Z}$. 40%

Section – B

Q4. **a).** If a SOP (sum of products) of a Boolean function $F(x, y, z)$ is given by 50%

$F(x, y, z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$

i). Construct the truth table for $F(x, y, z)$.

ii). Construct corresponding Karnaugh map for the above truth table. Indicate the SOP groups to minimize $F(x, y, z)$.

iii). Using Karnaugh map, find the minimized expression for $F(x, y, z)$.

b). Using laws of Boolean algebra, prove that 25%

$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz = \bar{x}\bar{y} + \bar{y}\bar{z} + \bar{x}\bar{z} + xyz.$

c). Using laws of Boolean algebra, minimize the following expressions. 25%

i). $F_1(x, y) = (\bar{x} + y)(\bar{x} + \bar{y})$

ii). $F_2(x, y, z) = \overline{(x + y)}z + \bar{x}yz + x\bar{y}$

Q5. **a).** Find the domain of each of the following real-valued functions: 30%

i). $f(x) = \dfrac{(x-1)(x-2)}{x^2+11x+30}$.

ii). $g(x) = \dfrac{x^2+25}{\sqrt{x^2-25}}$.

b). Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined as follows: 15%

$$f(x) = \begin{cases} x^2+2x-3, & \text{if } x \leq -3 \\ x^2+4, & \text{if } -3 < x \leq 5 \\ x^2-2x+3, & \text{if } 5 < x \end{cases}$$

Find the values of $f(-3)$, $f(0)$ and $f(6)$.

c). Let $A = \mathbb{R} - \left\{-\frac{3}{4}\right\}$ and $B = \mathbb{R} - \left\{\frac{5}{4}\right\}$. The function f is defined as follows. $f: A \to B$ and $f(x) = \dfrac{5x+7}{4x+3}$. 35%

i). Show that f is one to one function.

ii). Show that f is on to function.

iii). Find the inverse function of f.

d). The real valued functions f and g are defined as $f(x) = 2x+5$ and $g(x) = x^2+2x+5$. Find $f \circ g$ and $g \circ f$. 20%

Q6. **a).** Express the following sets in Roster form. 20%

i). $A = \{x: 8 + x = 5;\ x \in \mathbb{N}\}$.

ii). $B = \{x: 0 < |x-2| < 3, x \in \mathbb{Z}\}$

b). Express the following sets in Builder form. 20%

i). $X = \{-1, 1, 3, 5, 7, 9 \ldots\ldots\ldots\}$ with $\mathbb{N}$.

ii). $X = \{-1, 1, 3, 5, 7, 9 \ldots\ldots\ldots\}$ with $\mathbb{Z}^+$.

c). Let A, B and C be any subsets of a universal set S. 30%

i). Using the method of contradiction, prove that $(A - C) \cap (C - B) = \emptyset$

ii). Show that, $(B - A) \cup (C - A) = (B \cup C) - A$ without using Venn Diagrams.

d). Let $A = \{3, 5, 7\}$ and $B = \{2, 4\}$. 30%

i). Find $A \times B$ and $B \times A$.

ii). Let $R = \{(3,4), (5,4), (7,4)\}$. Is R a relation from A to B or B to A? Justify your answer.

Section – C

Q7. **a).** Find the complex roots of $\sqrt{9+40i}$. Hence, solve the quadratic equation $x^2+2ix-10-40i=0$. 40%

b). Let $P(x)=x^4-5x^3+7x^2-5x+6$. 25%

i). Show that $x+i$ and $x-i$ are factors of $P(x)$.

ii). Hence solve the equation $P(x)=0$.

c). Write down a quadratic equation of the form $ax^2+bx+c=0$, satisfying the three conditions, where a,b,c are non-zero, $b^2-4ac>0$ and the roots are imaginary numbers. Justify your answer. 35%

Hint: The roots of the quadratic equation of the form $ax^2+bx+c=0$ are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \text{where } a\neq 0.$$

Q8. **a).** Let $z,z_1,z_2\in\mathbb{C}$. Prove that the following results. 30%

i). $z\bar{z}=|z|^2$ ii). $\overline{z_1+z_2}=\bar{z_1}+\bar{z_2}$ iii). $\overline{z_1z_2}=\bar{z_1}\bar{z_2}$

vi). $z+\bar{z}=2Re(z)$ v). $\bar{\bar{z}}=z$ vi). $Re(-z)=-Re(z)$

b). Using the above results, prove that $|z_1+z_2|^2=|z_1|^2+|z_2|^2+2Re(z_1\bar{z_2})$. Deduce that $|z_1+z_2|^2+|z_1-z_2|^2=2(|z_1|^2+|z_2|^2)$. Find the geometrical interpretation of $|z_1+z_2|^2+|z_1-z_2|^2=2(|z_1|^2+|z_2|^2)$. 40%

c). The complex numbers $2\left(cos\frac{\pi}{12}+i\,sin\frac{\pi}{12}\right)$ and $2\left(cos\frac{5\pi}{12}+i\,sin\frac{5\pi}{12}\right)$ represents by the points P_1 and P_2 in an Argand plane. Show that OP_1P_2 is a equilateral triangle, where O is the origin of the Argand plane. 30%

Q9. **a).** Let $z=-12+5i$. Find the $|z|$, $Arg(z)$ and $arg(z)$. 20%

b). Prove that the roots of the equation $z^3=-8$ can be expressed as 30%

$$2(cos\pi+isin\pi), 2\left(cos\frac{\pi}{3}+isin\frac{\pi}{3}\right) \text{and } 2\left(cos\left(-\frac{\pi}{3}\right)+isin\left(-\frac{\pi}{3}\right)\right)$$

c). If $z_1=\dfrac{1+\sqrt{3}i}{\sqrt{3}+i}$ and $z_2=\dfrac{-1+3i}{1+2i}$, then express z_1,z_2 and $\dfrac{z_2}{z_1}$ of the form $a+ib$ and $r(cos\,\theta+i\,sin\,\theta)$, where $a,b\in\mathbb{R}$ and $r>0, -\pi<\theta\leq\pi$. 50%

Hence, prove that $sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$ and $cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}$.

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