THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics & Philosophy of Engineering

Bachelor of Software Engineering Honors

Final Examination (2023/2024)

MHZ4256: Mathematics for Computing

Date: 21ˢᵗ August 2024 (Wednesday)   **Time: 0930 hrs. – 1230 hrs.**

Instructions:

- *Answer six (06) questions by selecting at least one question from each of sections A, B and C.*

- *This paper consists of nine (09) questions.*

- *All the symbols are in standard notation unless they are defined.*

Section – A

Q1. **a).** State whether each of the following statements is a proposition or not. Explain   15%
the truth value of each.

    i). $\forall x \in \mathbb{R}^+(x < -4 \Rightarrow x^3 = 1000)$.

    ii). $\forall x \in \mathbb{R}(x = -y \Rightarrow x^2 y = 1000)$.

    iii). The function $f(x) = cos x$, where $x \in \mathbb{R}$, is a periodic function and π is an even number.

b). Find the converse, inverse and contrapositive propositions of the following   15%
proposition.

If the formula is correct, then the solutions are correct.

c). Let p and q be propositions.   55%

    i). Construct one truth table for the propositions $\neg p, \, p \vee q, \, (p \Rightarrow q)$, $p \wedge q, \, \neg p \vee q$ and $p \Leftrightarrow q$.

    ii). Show that $p \Leftrightarrow q$ is a logical consequence of $p \Rightarrow q$, $p \wedge q$ and $p \vee q$.

    iii). Show that $\big((p \Rightarrow q) \wedge (p \vee q)\big) \Rightarrow (p \Leftrightarrow q)$ is a tautology.

    iv). What is the relation between $p \Rightarrow q$ and $\neg p \vee q$?

d). Let p, q and r be propositions. By using laws of the algebra of propositions, prove   15%
that $(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \equiv p \vee q$.

Q2. **a).** Determine the truth values of the following statement and write down the   30%
negation of the following statement.

$$\forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \left((|x| > |y|) \Rightarrow (x^2 > y^2)\right)$$

b). Find the truth value of $(\forall x)\left(Q(x) \Rightarrow P(a,\ g(x))\right)$. Given that   30%

$$D = \{0,\ 1\},\ a = 1,\ g(0) = 1,\ g(1) = 0,\ Q(0) = F,\ Q(1) = T$$
$$P(0,\ 0) = T,\ P(0,\ 1) = T,\ P(1,\ 0) = F,\ P(1,\ 1) = T.$$

c). Using the method of direct proof, prove that for all $n \in \mathbb{Z}$, if $7n + 9$ is even,   20%
then n is odd.

d). By using the method of proof by contrapositive, prove that if $2|n^3$, then $2|n$.   20%

Q3. **a).** Let $x,\ y, \in \mathbb{R}$. Using the method of proof by contradiction, prove that   30%

$$\forall x \forall y \left(6x + 5y > 5 \Rightarrow \left(x > \frac{8}{7} \vee y > \frac{7}{15}\right)\right).$$

b). By using the principle of mathematical induction, prove that $8^{n+1} + 9^{2n-1}$ is   30%
divisible by 73.

c). Using the proof by case, show that $2n^2 + 5n + 4$ is not divisible by 3, when   40%
$n \in \mathbb{Z}$.

Section – B

Q4. **a).** If a SOP (sum of products) of a Boolean function $F(x, y, z)$ is given by   50%

$$F(x, y, z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

i). Construct the truth table for $F(x, y, z)$.

ii). Construct corresponding Karnaugh map for the above truth table.
Indicate the SOP groups to minimize $F(x, y, z)$.

iii). Using Karnaugh map, find the minimized expression for $F(x, y, z)$.

b). Using laws of Boolean algebra, prove that   25%

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz = \bar{x}\bar{y} + \bar{y}\bar{z} + \bar{x}\bar{z} + xyz.$$

c). Using laws of Boolean algebra, minimize the following expressions.   25%

i). $F_1(x, y) = (\bar{x} + y)(\bar{x} + \bar{y})$

ii). $F_2(x, y, z) = \overline{(x + y)z} + \bar{x}yz + x\bar{y}$

Q5. **a).** Find the domain of each of the following real-valued functions: 30%

i). $f(x) = \dfrac{(x-1)(x-2)}{x^2 + 11x + 30}$.

ii). $g(x) = \dfrac{x^2 + 25}{\sqrt{x^2 - 25}}$.

b). Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined as follows: 15%

$$f(x) = \begin{cases} x^2 + 2x - 3, & \text{if } x \le -3 \\ x^2 + 4, & \text{if } -3 < x \le 5 \\ x^2 - 2x + 3, & \text{if } 5 < x \end{cases}$$

Find the values of $f(-3)$, $f(0)$ and $f(6)$.

c). Let $A = \mathbb{R} - \left\{-\dfrac{3}{4}\right\}$ and $B = \mathbb{R} - \left\{\dfrac{5}{4}\right\}$. The function f is defined as 35%

follows. $f : A \to B$ and $f(x) = \dfrac{5x + 7}{4x + 3}$.

i). Show that f is one to one function.

ii). Show that f is on to function.

iii). Find the inverse function of f.

d). The real valued functions f and g are defined as $f(x) = 2x + 5$ and 20%

$g(x) = x^2 + 2x + 5$. Find $f \circ g$ and $g \circ f$.

Q6. **a).** Express the following sets in Roster form. 20%

i). $A = \{x : 8 + x = 5; \ x \in \mathbb{N}\}$.

ii). $B = \{x : 0 < |x - 2| < 3, x \in \mathbb{Z}\}$

b). Express the following sets in Builder form. 20%

i). $X = \{-1, 1, 3, 5, 7, 9 \ldots \ldots \ldots\}$ with $\mathbb{N}$.

ii). $X = \{-1, 1, 3, 5, 7, 9 \ldots \ldots \ldots\}$ with $\mathbb{Z}^+$.

c). Let A, B and C be any subsets of a universal set S. 30%

i). Using the method of contradiction, prove that $(A - C) \cap (C - B) = \emptyset$

ii). Show that, $(B - A) \cup (C - A) = (B \cup C) - A$ without using Venn Diagrams.

d). Let $A = \{3, 5, 7\}$ and $B = \{2, 4\}$. 30%

i). Find $A \times B$ and $B \times A$.

ii). Let $R = \{(3, 4), (5, 4), (7, 4)\}$. Is R a relation from A to B or B to A? Justify your answer.

Section – C

Q7. **a).** Find the complex roots of $\sqrt{9 + 40i}$. Hence, solve the quadratic equation   40%

$x^2 + 2ix - 10 - 40i = 0$.

b). Let $P(x) = x^4 - 5x^3 + 7x^2 - 5x + 6$.   25%

   i). Show that $x + i$ and $x - i$ are factors of $P(x)$.

   ii). Hence solve the equation $P(x) = 0$.

c). Write down a quadratic equation of the form $ax^2 + bx + c = 0$, satisfying the   35%
three conditions, where a, b, c are non-zero, $b^2 - 4ac > 0$ and the roots are
imaginary numbers. Justify your answer.

Hint: The roots of the quadratic equation of the form $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{where } a \neq 0.$$

Q8. **a).** Let $z, z_1, z_2 \in \mathbb{C}$. Prove that the following results.   30%

   i). $z\bar{z} = |z|^2$ **ii).** $\overline{z_1 + z_2} = \bar{z_1} + \bar{z_2}$ **iii).** $\overline{z_1 z_2} = \bar{z_1} \bar{z_2}$

   vi). $z + \bar{z} = 2Re(z)$ **v).** $\bar{\bar{z}} = z$ **vi).** $Re(-z) = -Re(z)$

b). Using the above results, prove that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z_2})$.   40%
Deduce that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$. Find the geometrical
interpretation of $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.

c). The complex numbers $2\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$ and $2\left(\cos\dfrac{5\pi}{12} + i\sin\dfrac{5\pi}{12}\right)$ 30%

represents by the points P_1 and P_2 in an Argand plane. Show that OP_1P_2 is a
equilateral triangle, where O is the origin of the Argand plane.

Q9. **a).** Let $z = -12 + 5i$. Find the $|z|$, $Arg(z)$ and $arg(z)$.   20%

b). Prove that the roots of the equation $z^3 = -8$ can be expressed as   30%

$$2(\cos\pi + i\sin\pi), 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \text{ and } 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

c). If $z_1 = \dfrac{1 + \sqrt{3}i}{\sqrt{3} + i}$ and $z_2 = \dfrac{-1 + 3i}{1 + 2i}$, then express z_1, z_2 and $\dfrac{z_2}{z_1}$ of the form   50%

$a + ib$ and $r(\cos\theta + i\sin\theta)$, where $a, b \in \mathbb{R}$ and $r > 0, -\pi < \theta \leq \pi$.

Hence, prove that $\sin\dfrac{\pi}{12} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$ and $\cos\dfrac{\pi}{12} = \dfrac{\sqrt{6} + \sqrt{2}}{4}$.

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