The Open University of Sri Lanka
B.Sc./B.Ed. Degree, Continuing Education Programme
No Book Test (NBT) - 2024/2025
Level 4 - Applied Mathematics
ADU4301 – Newtonian Mechanics I

Date: 09-03-2025 **Time: 9:00 a.m. To 10:00 a.m.**

Answer All Questions.

1. (a) With the usual notation show that the equation of the central orbit of a particle of unit mass moving in a plane is given by

$$\frac{d^2u}{d\theta^2}+u=\frac{F}{h^2u^2} \qquad \text{and} \qquad \dot{\theta}=hu^2 .$$

(b) A particle moves under the central force μ/r^3 per unit mass. It is projected from an apse at a distance a form the origin with velocity $\frac{2}{a}\sqrt{\frac{\mu}{3}}$. Show that the path is given by $r\cos(\theta/2)=a$.

2. Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}+\underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mas $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

A rocket has total mass M. It propels itself by burning fuel and ejecting the burnt matter at a uniform rate with constant speed u relative to the rocket. The total mass of the fuel in the rocket is initially $\frac{M}{2}$ and the fuel is all burnt up after a time T.

The rocket is launched from rest vertically upwards from the surface of the Earth. It may be assumed that the acceleration due to gravity remains constant throughout the flight of the rocket, and that air resistance is negligible. At time t the speed of the rocket is v.

(a) Show that, while the fuel is being burnt (t<T), $(2T-t)\frac{dv}{dt}=u-g(2T-t)$.

(b) **Hence**, find the speed of the rocket at the instant when all the fuel has been burnt.