The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

No Book Test 2024/2025

Level 03 Pure Mathematics

PEU3202/PEE3202– Vector Spaces

Duration: - One hour

Date: -14-03-2025 **Time: 9.00 -10.00 a.m.**

Answer all questions

1.

(a) Let $M = \{(a,b,c,d) \mid a,b,c,d \in \mathbb{R}\}$. Note that M is a vector space over the field $\mathbb{R}$ under the usual addition and scalar multiplication.
Let the mapping $T : M \to M$ be defined by $T((a,b,c,d)) = (a+b, b, 3c, c+d)$.
Note that T is a linear Transformation. Determine whether the following sets are invariant subspaces of the vector space M over the field $\mathbb{R}$ under T

(i) $W = \{(a,b,0,0) \mid a.b \in \mathbb{R}\}$

(ii) $W = \{(a,0,0,b) \mid a,b \in \mathbb{R}\}$

(b)

(i) Define an inner product space.

(ii) Let V be an inner product space over a field F. Prove that for $x_1, x_2, y_1, y_2 \in V$,
$< x_1 + x_2, y_1 + y_2 > = < x_1, y_1 > + < x_1, y_2 > + < x_2, y_1 > + < x_2, y_2 >$

(iii) Let $u = (x_1, x_2, x_3)$, $v = (y_1, y_2, y_3)$ where $u, v \in \mathbb{R}^3$.

Define $< u, v > = {x_1}^2 + {x_2}^2 + {x_3}^2$. Is $< u, v >$ an inner product on $\mathbb{R}^3$? Justify your answer.

2.

(a) Let u and v be any two vectors of a Euclidian Space.

(i) Define the length of u and the distance between u and v.

(ii) Suppose E^3 be the usual Euclidian three space. Let $u = (1, -1, 2)$ and $v = (2,1,0)$. Find the length of u and the distance between u and v.

(b) Show that the three vectors $u_1 = (0,1,1)$, $u_2 = (0,0,1)$ and $u_3 = (1,0,-1)$ form a basis for E^3, the usual Euclidean three space. Construct an orthogonal basis for E^3 out of $\{u_1, u_2, u_3\}$ using the Gram–Schmidt process.