The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Open Book Test 2024/2025

Level 03 Pure Mathematics

PEU3202– Vector Spaces

Duration: - One hour

Date: -09-02-2025 **Time: 4.00 -5.00 p.m.**

Answer all questions

1.

(a) Let $V = \{ (a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$. For every $(a_1, a_2), (b_1, b_2) \in V$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$ for $c \in \mathbb{C}$ where $\mathbb{C}$ is the complex number field. Is V a vector space over the field of complex numbers under these operations? Justify your answer.

(b) Let $V = \{ (a_1, a_2) \mid a_1, a_2 \in \mathbb{R}\}$ For every $(a_1, a_2), (b_1, b_2) \in V$ define $(a_1, a_2) + (b_1, b_2) = (2a_1 + b_1, a_2 + 3b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$ for $c \in \mathbb{R}$ where $\mathbb{R}$ is the field of real numbers . Is V a vector space over the field of real numbers under these operations? Justify your answer.

(c) Determine whether the set $A = \{(a + 2b, a + 1) \mid a, b \in \mathbb{R}\}$ is a subspace of the vector space $\mathbb{R}^2$ over the field $\mathbb{R}$ under usual addition and scalar multiplication.

(d) Let $S = \{P_1 = 1 - x, P_2 = 5 + 3x - 2x^2, P_3 = 1 + 3x - x^2\}$ be a sub set of the vector space of all polynomials of degree at most 2 over $\mathbb{R}$. Is S linearly independent over the field $\mathbb{R}$? Justify your answer.

2.

Let $M = \{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \}$. Note that M is a vector space over the field $\mathbb{R}$ under the usual matrix addition and scalar multiplication.

Let the mapping $T : M \to M$ be defined by $T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} a + d & b + c \\ c & c + d \end{bmatrix}$.

(i) Show that T is a linear transformation

(ii) Find the kernel of T.

(iii) Is T an isomorphism? Justify your Answer.