THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./ B.Ed. Degree Programme
APPLIED MATHEMATICS - LEVEL 05
ADU5307 - Numerical Methods
Final Examination - 2024/2025
DURATION: Two Hours

Date: 26.11.2024 **Time: 1.30 p.m. - 3.30 p.m.**

Answer Four Questions Only.

1. (a) (i) Derive Newton- Raphson formula for solving the equation $f(x) = 0$.

(ii) Show that the equation $x^3 + 2x - 2 = 0$ has a root in the interval $[0, 1]$ and use method of the Newton-Raphson to find the root correct to two decimal places near to the point $x_0 = 0.5$.

(b) (i) Suppose that equation $f(x) = 0$ has a solutions in the interval $[a, b]$. If the permissible error is ϵ, then determine the approximate number of iterations required?

(ii) For the given function $f(x) = x^3 - 4x + 3$, a real root lies in between the interval $[-3, -2]$. Find the minimum number of iterations required to find the root up to the accuracy of three decimal points.

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2. (a) Prove that
(i) $\Delta = E - I$,
(ii) $\delta^2 = \Delta - \nabla$,
(iii) $E^{\frac{1}{2}} = \mu + \frac{1}{2}\delta$.
where $\Delta, \nabla, \mu, E, I$ and δ are the forward difference, the backward difference, average , the shift, identity and the the central difference operators respectively.

(b) (i) Derive Gregory - Newton forward interpolation polynomial for $y = f(x)$ for equidistance intervals.

(ii) A school records the number of students enrolled over several years. The table below shows the year (in terms of an index x) and the corresponding number of students enrolled in that year, $y(x)$.

Year (Index x)	40	50	60	70	80
No. of Students y(x)	31	73	124	159	190

Using Gregory - Newton forward interpolation formula, estimate the number of students enrolled in the year corresponding to an index value 45.

3. (a) Derive the Lagrange's interpolation formula for $y = f(x)$ for unique intervals.

(b) The following table shows the values of $y = ln(x)$, where x is a measurement of concentration in a chemical solution:

x	1	2.5	3
y = ln(x)	0	0.9163	1.0986

Find the Second-order polynomial to the above data points by applying Lagrange's interpolation formula and hence determine the value of $ln(2.7)$ correct to four decimal places.

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4. (a) Using the Newton's forward difference formula, derive the Trapezoidal Rule for the function $y = f(x)$ define over the equidistance intervals $(x_0, x_1, x_2, ..., x_n)$.

(b) The velocity of a particle which starts from rest is given by the following table.

Time (s)	0	2	4	6	8	10	12
Velocity (ms^{-1})	0	16	29	40	46	51	57

Evaluate the total distance traveled in 12 seconds using,

(i) Trapezoidal rule.

(ii) Simpson's One –Third Rule rule.

(c) Applying Taylor series method of third order for the differential equation $y'' = y + xy'$ subject to the initial conditions $y(0) = 1$, and $y'(0) = 0$, evaluate $y(0.1)$ correct to three decimal places.

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5. (a) Consider the following initial value problem

$$y'(t) - \frac{1}{2}y = 1, \quad y(0) = 1.$$

Find the approximate solution for $y(0.2)$ correct to four decimal places by taking the step size $h = 0.1$ using

(i) Euler's method.

(ii) modified Euler's method.

(iii) What can say about the accuracy of the solution of the given initial value problem?

(b) (i) Derive the formula for Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first- two successive approximations to solve $\frac{dy}{dx} = 2(y + 1)$ with the initial condition $y(0) = 0$.

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6. (a) (i) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using fourth order Runge-Kutta algorithm, solve $\frac{dy}{dx} = 1 + y$ correct to four decimal places subject to the initial condition $y(0) = 0$, at $x = 0.1$.

(b) Given $\frac{dy}{dx} = 1+y$, $y(0) = 0$, $y(0.1) = 0.1052$, $y(0.2) = 0.2214$ and $y(0.3) = 0.3499$. Estimate $y(0.4)$, by using Adam-Bash forth formula.

$$[Hint : y_{n+1,p} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3})$$

$$y_{n+1,c} = y_n + \frac{h}{24}(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2})]$$

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