The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed. Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Linear Programming - ADU5300**
Academic Year	**: 2024/25**
Date	**: 11.12.2024**
Time	**: 01.30 p.m. –03.30 p.m.**
Duration	**: Two Hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(5)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Involvement in any activity that is considered as an exam offense will lead to punishment.
6. Use blue or black ink to answer the questions.
7. Clearly state your index number in your answer script.

Answer any **FOUR** questions only

1. A firm makes two products P1 and P2 and the firm has a total production capacity of 18 tons per day. P1 and P2 require same production capacity. The firm must supply at least 4 tons of P1 and 6 tons of P2 per day. Each ton of P1 and P2 requires 60 hours of machine work. The maximum machine hours available is 720. Profit per ton for P1 is Rs. 160 and P2 is Rs. 240.

   i) Formulate a linear programming model, by clearly defining the decision variables, to maximize the total profit. (08 marks)

   ii) Determine graphically the feasible region. (07 marks)

   iii) Show that the feasible region determined in (ii) is a convex set. (06 marks)

   iv) Find the optimal solution using the graphical method. (04 marks)

[Total marks 25]

2. a) Solve the following linear programming problem using the **revised simplex method**:

$$
\begin{aligned}
&\textit{Maximize } z = 5x + 8y \\
&\textit{s.t.} \quad 3x + 2y \le 36 \\
&\qquad x + 2y \le 20 \\
&\qquad 3x + 4y \le 42 \\
&\qquad x, y \ge 0
\end{aligned}
$$

(15 marks)

b) Consider the following linear programming problem:

$$
\begin{aligned}
&\textit{Maximize } z = x_1 + 2x_2 + x_3 \\
&\textit{s.t.} \quad 2x_1 + x_2 - x_3 \le 2 \\
&\qquad 2x_1 - x_2 + 5x_3 \le 6 \\
&\qquad 4x_1 + x_2 + x_3 \le 6 \\
&\qquad x_1, x_2, x_3 \ge 0
\end{aligned}
$$

Apply the simplex method, while incorporating the degeneracy procedure to avoid cycling, to solve the given problem.

(10 marks)

[Total marks 25]

3. Consider the following linear programming problem:

$$\begin{aligned} &\textit{Maximize } z = x_1 + x_2 \\ &s.t. \quad x_1 - x_2 - x_3 = 1 \\ &\quad -x_1 + x_2 + 2x_3 - x_4 = 1 \\ &\quad x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

a) Solve the given problem using the Big- M method. (15 marks)

b) Solve the same problem using the Two-phase simplex method.
Explain the advantage of using the Two-phase simplex method over to Big-M method to solve this problem. (10 marks)

[Total marks 25]

4. a) Write the dual problem of the following primal problem:

$$\begin{aligned} &\textit{Maximize } z = 5x_1 + 6x_2 \\ &s.t. \quad x_1 + 2x_2 = 5 \\ &\quad -x_1 + 5x_2 \geq 3 \\ &\quad 4x_1 + 7x_2 \leq 8 \\ &\quad x_1 - \textit{unrestricted } x_2 \geq 0 \end{aligned}$$

(10 marks)

b) Solve the following linear programming problem using the dual simplex method.

$$\begin{aligned} &\textit{Minimize } z = 2x_1 + x_2 \\ &s.t. \quad x_1 \geq 2 \\ &\quad 3x_1 + 4x_2 \leq 24 \\ &\quad 4x_1 + 3x_2 \geq 12 \\ &\quad -x_1 + 2x_2 \geq 1 \\ &\quad x_1, x_2 \geq 0 \end{aligned}$$

(15 marks)

[Total marks 25]

5. A department of a company has five employees to perform five jobs. The time (in hours) that each employee takes to perform each job is given in the following effectiveness matrix:

Job \ Employee	I	II	III	IV	V
A	10	5	13	15	16
B	3	9	18	13	6
C	10	7	2	2	2
D	7	11	9	7	12
E	7	9	10	4	12

i) Formulate a mathematical model, by clearly defining the decision variables, to determine the optimal assignment. (05 marks)

ii) Use **Hungarian Algorithm** to find the optimal assignment that minimizes the total man-hours. (15 marks)

iii) Determine the employee assigned to the respective job. (03 marks)

iv) Find the minimum time required to perform all the jobs. (02 marks)

[Total marks 25]

6. i) Explain degeneracy in a transportation problem? (05 marks)

ii) A company has three factories, F1, F2, and F3 and three warehouses, W1,W2, and W3. The shipping cost (in thousand rupees) from factories to the warehouses are shown in the table below:

	W1	**W2**	**W3**	**Factory capacity in units**
F1	5	4	3	250
F2	8	4	3	300
F3	9	7	5	300
Warehouse requirement in units	300	200	200	

a) Formulate the above problem as a transportation problem in order to minimize the transportation cost. (03 marks)

b) Find the initial basic feasible solution (IBFS) using the North-West corner method. (05 marks)

c) Using the IBFS obtained in b), apply the transportation algorithm to find the optimal manufacturing cost. (10 marks)

d) Determine the minimum transportation cost. (02 marks)

[Total marks 25]

********* END OF QUESTION PAPER *********