The Open University of Sri Lanka
B.Sc/ B.Ed. Degree Programme
Final Examination - 2024/2025
Applied Mathematics - Level 05
ADU5308 – Graph Theory

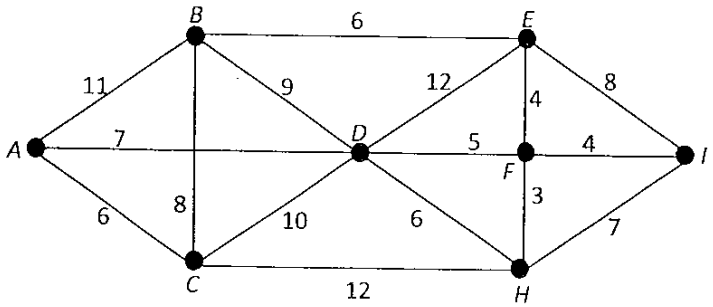
Duration: Two hours

Date: 15-05-2025 **Time: 09.30 a.m. – 11.30 a.m.**

Answer **four** questions **only.**

1.

a) Define an **Eulerian Graph.**
Solve the Chinese Postman problem for the flowing weighted graph G.



G

b)

i. Evaluate $+\uparrow 7-5\ 3*8\ 2$ (Polish).

ii. Draw the corresponding parse tree for the mathematical expression $5(x^4-2y)-2(y^3+x)$.

c) Consider the following lexicographic ordering of the labelling:
$0 > 1 > 1.1 > 1.1.1 > 1.1.2 > 1.2 > 1.2.1 > 1.2.2 > 2 > 2.1 > 2.2 > 3 > 3.1 > 3.1.1 > 3.1.2 > 3.1.2.1 > 3.1.2.2 > 4 > 4.1 > 4.2 > 4.2.1 > 4.2.2 > 5 > 5.1 > 5.2 > 5.3$

i. Draw the labelling of the universal addresses system next to the vertices in the ordered rooted tree.

ii. Write down the in-order traversal for the rooted tree in part (i).

2.

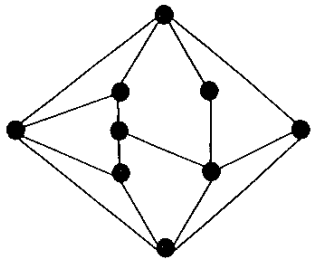
a) Suppose that a chemist wishes to store six chemicals 1, 2, 3, 4, 5, and 6 in various areas of a laboratory. Some of these chemicals react violently when in contact and so must be kept in separate areas. In the following table, an asterisk indicates those pairs of chemicals that must be separated.

	1	2	3	4	5	6
1		*		*	*	*
2	*		*		*	
3		*		*	*	*
4	*		*			*
5	*	*	*			
6	*		*	*		

i. Draw the corresponding weighted graph for the above table.
ii. Find the minimum number of areas needed to store chemicals.

b)

i. Define the **Chromatic Number** and **Chromatic Index** for a graph G.
ii. Find the chromatic number and chromatic index of the given graph G.



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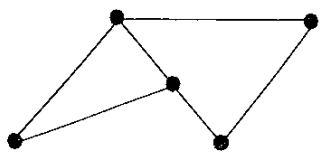
iii. Is the graph G satisfy the **Four-color theorem** or not? Justify your answer.

3.

a) Consider the flowing graph G and construct the followings:



G

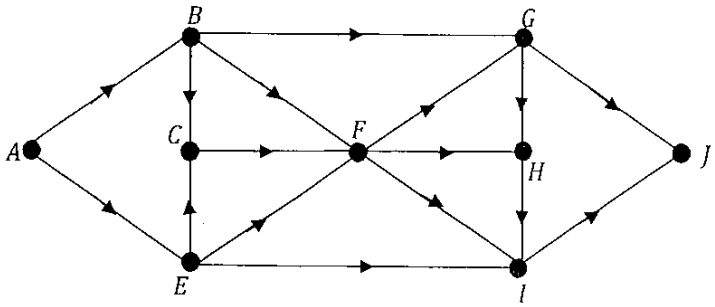
i. Subdivision graph $S(G)$.
ii. 2nd iterated line graph of G, $L^2(G)$.
iii. Line graph $L_2(G)$.

b)

i. Let T be a graph with n vertices. If T is a tree, then prove that T contains no cycles and has $n-1$ edges.
ii. How many edges must be removed from a connected graph with n vertices and m edges to produce a spanning tree?

4.

a) State the **Handshaking Dilemma**.
Consider the digraph $D = (V, A)$.



D

i. Find $d(A, a)$ for all $a \in V$.
ii. Find $d(a, J)$ for all $a \in V$.
iii. The digraph D is strong or not. Justify your answer.

b) The results of a tournament between five teams a, b, c, d, and e are given in the following table.

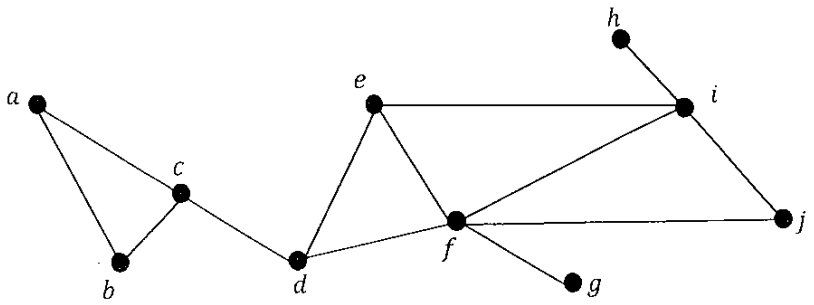
Match	Winner
a vs b	a
a vs c	c
a vs d	a
a vs e	e
b vs c	b
b vs d	d
b vs e	b
c vs d	d
c vs e	c
d vs e	e

i. Construct the digraph according to the results of the given tournament.
ii. Find cycles of length 4 and 5.
iii. Find a closed Eulerian trail.
iv. Find a Hamiltonian cycle.

c) In the tournament given in part (b), verify that $\sum \text{in deg}(v)^2 = \sum \text{out deg}(v)^2$.

5.

a) Define the terms **Cut point**, **Bridge**, and **Block** of a graph.
Consider the following graph G :
i. Find all the cut point(s), bridge(s), and block(s) of the graph G.
ii. Construct the corresponding block graph and cut point graph for the graph G.



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b) Define a **Complete Matching**.
Suppose that four girls a, b, c, d know seven boys t, u, v, w, x, y, z as given in the following table.

Girl	Boys known by girl
a	t w x
b	u v w z
c	x y z
d	t w

i. Draw the bipartite graph corresponding to these relationships.
ii. Check the marriage condition for this problem.

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6.

a) Consider the family $\mathfrak{I} = (S_1, S_2, S_3, S_4, S_5, S_6)$ of subset of $E = \{1, 2, 3, 4, 5, 6\}$, where $S_1 = S_3 = \{1, 3\}$, $S_2 = \{3, 4\}$, $S_4 = \{2, 3, 5\}$ and $S_5 = S_6 = \{1, 2, 4, 6\}$.

i. Obtained the corresponding incident matrix.

ii. Find the term rank.

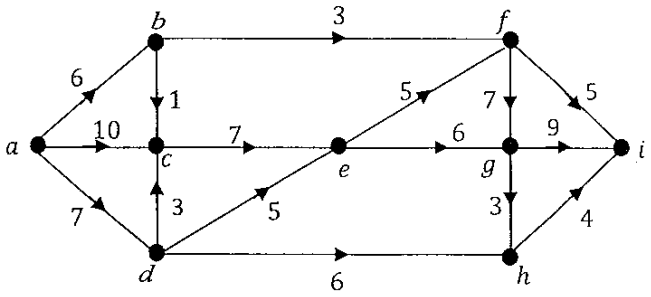
iii. State **Konig-Egervary theorem** and verify it for this problem.

iv. Determine whether the above family has transversals or not. Justify your answer.

b) State the **Max-flow, Min-cut theorem**.

i. Verify Max-flow, min-cut theorem for the following Network N.

ii. Identify the saturated arcs for the Network N.



N

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