The Open University of Sri Lanka
B.Sc./ B.Ed. Degree Programme
Final Examination - 2024/2025
Applied Mathematics - Level 05
ADU5306 - Fluid Mechanics
DURATION: TWO HOUR

Date: 22.05.2025 **Time: 9.30 a.m. to 11.30 a.m.**

Answer **FOUR** questions only.

1. (a) By considering the equilibrium of an arbitrary volume V of fluid at rest, derive the equation of equilibrium in the form $\rho \underline{F} = grad\, p$.

   (b) A mass of incompressible gas extends from infinity to the surface of a rigid sphere $r = a$.The temperature being constant, the pressure p and density ρ of the gas are related by an equation of the form $p = K\rho$ where K is a constant.The external force per unit mass at a point P is given by $\underline{F} = -\frac{\mu \underline{r}}{r^3}$, where μ is a positive constant and $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ is the position vector of point P with respect to O, its magnitude being denoted by r. Show that the pressure exerted on the surface of the sphere is $p = p_o e^{\frac{\mu}{Ka}}$; where p_o is the pressure at infinity.

2. (a) Derive the continuity equation of the form $\frac{D\rho}{Dt} + \rho\, div(\underline{q}) = 0$, for any arbitrary control volume of a moving fluid irrespective of its shape and size. $\underline{q}$ and ρ fluid velocity and fluid density respectively.

   (b) Hence deduce the continuity equation. for an incompressible fluid in terms of Cartesian Coordinate.

   (c) Consider the fluid flow field which is given by $\underline{q} = x^2y\underline{i} + y^2z\underline{j} - (2xyz + yz^2)\underline{k}$. Prove that this is a case of a posiible incompressible flow field.

   (d) Given $v = 2y^2$ and $w = 2xyz$, the two velocity components. Determine the third component such that it satisfies the continuity equation.

3. (a) Given Euler's equation of motion $\underline{F} - \frac{1}{\rho}\, grad\, p = \frac{D\underline{q}}{Dt}$ for a perfect fluid, show that it can be written in the form $\underline{F} - \frac{1}{\rho}\, grad\, p = \frac{\partial \underline{q}}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{q} \times curl\underline{q}$.

   (b) Using the result in Part (a), derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

(c) Consider a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 500mm^2 and the exit has a bore area of 250mm^2 . Assuming there is no energy loss calculate the flow rate when the inlet pressure is 400 Pa .

4. A stream of liquid has velocity V at infinity in the negative $x-$direction, the sphere $r = 2$, being a rigid boundary. Moreover the velocity potential of the flow is given by $\phi = V\left(r + \frac{4}{r^2}cos\theta\right)$.

   (a) Derive the components of velocity and hence obtain the stream function. $\left(\text{Hint: } -\frac{\partial\phi}{\partial r} = q_r = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} \text{ and } -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = q_\theta = \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial r}\right)$.

   (b) Find thee equation of a streamline which is at a distance a from the axis, at infinity, and show that such a streamline meets the plane $\theta = \frac{\pi}{2}$, at a point where $r = b$, given by $\left(b^2 - \frac{8}{b}\right) = a^2$.

5. The complex potential of a fluid flow is given by $W(z) = U\left(z + \frac{4}{z}\right)$ where U is positive constant.

   (a) Obtain the equation for the streamlines and velocity potential lines and represent them graphically.

   (b) Find the complex velocity at any point and determine its value far from the origin.

   (c) Find the stagnation points of the flow.

6. The components of fluid velocity $\underline{q}$, in terms of cylindrical polar coordinates r, θ, z are $-U\left(1 - \frac{a^2}{r^2}\right)\cos\theta$, $U\left(1 + \frac{a^2}{r^2}\right)\sin\theta$, 0 respectively.

   (a) Verify that this velocity represents irrotational motion of an incompressible fluid, and find the velocity potential.

   (b) Describe the behaviour of velocity on the surface of the cylinder $r = a$ and at infinity.

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