THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
FINAL EXAMINATION 2024/2025
APPLIED MATHEMATICS-LEVEL 05
ADU5304- OPERATIONAL RESEARCH
DURATION: TWO HOURS

Date: 16.05.2025	**Time: 09.30 a.m- 11.30 a.m**

ANSWER FOUR QUESTIONS ONLY.

Question 01

(a) (i) Define a game in Game Theory.
(ii) State the assumptions for a competitive game.
(iii) Explain the following terms: pay-off matrix, saddle point, pure strategy and mixed strategy.

(b) Solve the game with the following pay-off matrix:

Player *B*

Player *A*		I	II	III	IV
	1	1	-6	8	4
	2	3	-7	2	-8
	3	5	-5	-1	0
	4	3	-4	5	7

Question 02

(a) (i) Describe a *two-person zero sum game*.
(ii) State the assumptions for a two-person zero sum game.
(iii) Explain Minimax and Maximin principles adapted in Game Theory.

[Turn over

(b) There are two players in a game, player A and player B. Each player randomly shows selected fingers of his right hand. If the sum of the number of fingers shown by both the players is an even number, then the player B has to give money in rupees equivalent to the number of fingers shown by him to the player A; if the sum of the number of fingers shown by both the players is an odd number, then the player A has to give money in rupees equivalent to the number of fingers shown by him to the player B.

(i) Construct the payoff matrix with respect to player A and hence, solve the game.

Question 03

(a) Briefly explain the following terms in Queuing Theory:

(i) Arrival pattern

(ii) Service pattern

(iii) Queue discipline

(b) Assume a single-channel service system in a school library with Poisson arrivals and Exponential service. From the past experience, it is known that on average, 8 students come every hour to borrow books, and the rate of issuing books is 10 per hour. Determine the following:

(i) probability of the Library Assistant, who is responsible for issuing books, being idle.
(ii) probability that there are at least 3 students in the system.
(iii) expected waiting time that a student is in the queue.
(iv) probability that a student arriving at the library will have to wait before being served.

Question 04

Suppose people arrive to purchase tickets for a basketball game at an average rate of 4 per minute, and it takes 10 seconds, on average, to purchase a ticket. Assume that there is one counter to issue tickets, with Poisson arrivals and Exponential service.

If a person arrives just 2 minutes before the game starts, it takes exactly 1.5 minutes to be seated after purchasing a ticket.

(i) Can the person expect to be seated for the start of the game? Justify your answer.

(ii) What is the probability that the person will be seated before the game starts?

(iii) How early must the person arrive in order to be 99% certain of being seated for the start of the game?

Question 05

(a) (i) Define the term "Inventory".

(ii) What are the advantages and disadvantages of keeping inventories?

(iii) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and instantaneous supply.

(b) An industry estimates that it will sell 12,000 units of its product for the forthcoming year. The ordering cost is Rs.100 per order and the carrying cost per unit per year is 20% of purchase price per unit. The purchase price per unit is Rs. 50. Assumptions have to be made. Find the

(i) Economic Order Quantity,

(ii) number of orders per year,

(iii) time between successive orders,

(iv) total inventory cost per year.

Question 06

(a) Establish the Economic Order Quantity (EOQ) model in which demand is uniform and replenishment rate is finite ?.

(b) A unit is used at the rate of 100 per day and can be manufactured at a rate of 600 per day. It costs Rs. 2000 to set up the manufacturing process. and the holding cost is Rs. 0.1 per unit per day. Shortage is not allowed. Find the minimum cost and the optimum number of units per manufacturing run. Using the model established in part (a), determine the following:

(i) optimum number of units per manufacturing run.

(ii) time of the cycle,

(iii) minimum total inventory cost per manufacturing run.

Formulas (in the usual notation)

(M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq n) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad E(v) = \frac{1}{\mu - \lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu - \lambda)}$$

(M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1 - N\rho^{N-1} + (N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(n) = \frac{\rho\left[1 - (N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1 - P_N)$$

(M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ; 1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ; n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\dfrac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu - \lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu - \lambda}\right]^{-1} \qquad E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

(M/M/C): (N/FIFO) Model

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;0 \le n \le C \\ \dfrac{1}{C^{n-1}C!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;C < n \le N \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=C}^{\infty}\frac{1}{C^{n-C}C!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(m) = \frac{P_o(C\rho)^C\rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1}-(1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0\sum_{n=0}^{C-1}\frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

(M/M/R):(K/GD) Model

$$P_n = \begin{cases} \dbinom{K}{n}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;\ 0 \le n < R \\ \dbinom{K}{n}\dfrac{n!}{R^{n-R}R!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ;\ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda[K-E(n)]}$$

$$E(m) = \sum_{n=R}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda[K-E(n)]}$$

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