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The Open University of Sri Lanka

Faculty of Natural Sciences

B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title Code	**: Newtonian Mechanics II**
Course Title Code	**: ADU5303**
Academic Year	**: 2024/2025**
Date	**: 14.05.2025**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(2)** pages.
3. Answer any **Four (4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
6. Involvement in any activity that is considered as an exam offense will lead to punishment.
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script.

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1. (a) In the usual notation, show that in spherical polar coordinates, the

velocity $\underline{v}$ and $\underline{a}$ acceleration of a particle are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + r\dot{\phi}\sin\theta\,\underline{e}_\phi$ and

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 \sin^2\theta)\underline{e}_r + \left(\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) - r\dot{\phi}^2 \sin\theta\cos\theta\right)\underline{e}_\theta + \left(\frac{1}{r\sin\theta}\frac{d}{dt}(r^2\sin^2\theta\,\dot{\phi})\right)\underline{e}_\phi.$$

(b) A particle moves on a smooth sphere under no forces except for the reaction on the surface. Initially it is released from a point on a surface of the sphere with coordinates $(a, \beta, 0)$. Show that its path is given by the equation $\cot\theta = \cot\beta\cos\phi$ where θ and ϕ are its angular coordinates.

2. One end of an inextensible string of length l is attached to a fixed-point O on a ceiling and the other end attached to the end A of a uniform rod AB of length $2a$, the rod being suspended by the string. If both the rod and the string revolve about the vertical with uniform angular velocity ω , and if their inclinations to the vertical are θ and ϕ respectively, using D'Alembert's principle show that

$$\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}.$$

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 r}{\partial t^2} + 2\underline{\omega} \times \frac{\partial r}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) A projectile located at a point of latitude λ is projected with speed v_0 in a southward direction at an angle α to the horizontal. Find the position of the projectile after time t. Prove that after time t, the projectile will be deflected towards the east of the original vertical plane of motion by the amount

$$\frac{1}{3}\omega g\cos\lambda t^3 - \omega v_0 \sin(\alpha + \lambda)t^2.$$

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1,2 \ldots, n.$$

(b) A homogeneous rod OA of mass m_1 and length $2a$ is freely hinged at O to a fixed point. Another homogeneous rod AB of mass m_2 and length $2b$ is freely hinged at A to the free end of the rod OA. If the system moves in a fixed vertical plane, obtain equations of motion of the two rods, using Lagrange's equations. Find the vertical and horizontal components of the reaction at O..

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) The principal moments of inertia of a body at the centre of mass are A, $3A$, $6A$. The body is so rotated that its angular velocities about the axes are $3n$, $2n$, n respectively. If in the subsequent motion under no force, ω_1, ω_2, ω_3 denote the angular velocities about the principal axes at that time t, show that

$\omega_2 = 3n \ tanh(u)$ and $\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}}$ sech (u) where

$u = 3nt + \frac{1}{2} \ log_e 5$

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$ $i = 1,2 \ldots, n.$

(b) The Hamiltonian of a dynamical system is given by $H = qp^2 - qp + bp$ where b is a constant. Obtain Hamilton's equations of motion and hence find p and q at time t.