The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme -2009/2010

Final Examination

Pure Mathematics

Real Analysis-PMU 2193/PME 4193

Duration: Two and Half Hours

Date: 22/12/2009 **Time:1.00-3.30 p.m.**

Answer Four Questions Only.

1.(a) Show that $\sqrt{2}+5$ is an irrational number.

(b) If $\{a_n\}_{n=1}^{\infty}$ is strictly decreasing sequence, prove that $\{\sigma_n\}_{n=1}^{\infty}$ is also strictly decreasing where $\sigma_n = \frac{1}{n}\sum_{k=1}^{n} a_k$ for all $n = 1,2,3,\cdots$.

2 (a) Show that every increasing sequence which is bounded above, has a limit.

(b) Prove that the sequence whose general term is a_n given by $a_n = \sum_{k=0}^{n} \frac{1}{k!}$ is convergent

3. (a) Let A be a non-empty bounded subset of $\mathbb{R}$. If B is a non-empty subset of A, prove that

$$\text{Inf } A \leq \text{Inf } B \leq \text{Sup } B \leq \text{Sup } A .$$

(b) Find each of the following, using theorems on limits:

(i) $\lim_{n\to\infty}\left(2^n+3^n\right)^{\frac{1}{n}}$,

(ii) $\lim_{n\to\infty} x_n$, where $x_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}$ for $n = 1,2,3,\cdots$.

4. (a) Show that every convergent sequence is Cauchy.

(b) Show that the sequence $\{x_n\}_{n=1}^{\infty}$, where $x_n = \sum_{k=1}^{n} \frac{1}{k^2}$ is Cauchy.

5. (a) Show that an absolutely convergent series is convergent.

(b) Is the converse of the above theorem true? Justify your answer.

(c) Show that the following series are convergent and find their sums.

(i) $\sum_{n=1}^{\infty} \frac{5^n + 4^n}{6^n}$,

(ii) $\sum_{n=1}^{\infty} \frac{4^{2n-1}}{3^{3n+1}}$.

6. (a) Define the following phrases carefully:

(i) Deleted neighborhood of a point x in $\mathbb{R}$.

(ii) f is a continuous real-valued function on the closed interval $[a, b]$.

(b) Let $f: \mathbb{R} \to \mathbb{R}$ where

$$f(x) = \begin{cases} ax^2 + bx + 2, & x \in (-\infty, -1) \\ 2ax^2 + bx, & x \in [-1, 1] \\ \frac{2a}{x} - bx - 2, & x \in (1, \infty) \end{cases}.$$

If f is continuous on $\mathbb{R}$, find the values of a and b.