The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2009/2010
Pure Mathematics
PMU 2191/PME 4191 – **Vector Analysis**

Duration :- Two and Half (2 ½) Hours.

Date :- 05-01-2010. **Time:- 1.00 p.m. – 3.30 p.m.**

Answer Four Questions Only.

1. (a) If $\sin u = \dfrac{x^2+y^2}{x+y}$, show that

   (i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$ and

   (ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \tan^3 u$.

   (b) Using linear approximation obtain an approximate value for the real number, $7.96\sqrt{9.99-(1.01)^2}$.

   (c) Find the Taylor's expansion of $f(x, y) = e^y \ln(1+x)$ about the point (0, 0), up to 2nd order terms.

2. (a) (i) Define a stationary point of a single valued function $f(x, y)$, defined over the domain D. Explain briefly how you would determine its nature.

   (ii) Find the maximum and minimum values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ and determine their nature.

   (b) It is given that the directional derivative of $f(x, y, z) = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a maximum of magnitude 64 in a direction parallel to z- axis. Show that $a = 6$, $b = 24$ and $c = -8$.

   (c) The surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. Find the constants a and b.

3. **(a)** Find the value of the surface integral of the function $f(x,y) = y^2/x^2$ over the region bounded by $y = x,\ x = 1$ and $y = 2$.

(b) Evaluate $\int_S e^{x^2+y^2}\,dA$, where S is finite region in the 1st quadrant bounded by $x^2 + y^2 = a^2,\ y = 0,$ and $x = 0$.

(c) Find the volume integral of the function $f(x,\ y,\ z) = x^2 + y^2$ over the region bounded by the surfaces $z = x^2 + y^2,\ x = 0,\ \ y = 0$ and $z = 1$.

4. Let $U = x^2 + y^2 + z^2$ be a scalar field and S be the finite (closed) surface of the cylinder bounded by $x^2 + y^2 = a^2,\ \ z = 0,$ and $z = h$. Compute Surface integral $\int_S U\underline{n}\,dS$ over the cylinder where $\underline{n}$ is a unit vector normal to the surface S.

Hence verify that $\int_S U\underline{n}\,dS = \int_V grad\,U\,dV$ where V is the finite volume enclosed by S.

5. **(a)** State Gauss' divergence theorem, stating the meanings of any symbols used..

(b) Verify Gauss' divergence theorem for the vector field $\underline{F} = z^3\underline{k}$, considering the region enclosed by the surface S of a sphere of radius R with centre at the origin.

(c) Prove that **(i)** $\underline{\nabla}\cdot\underline{r} = 3$, (ii) $\underline{\nabla}\cdot\left(\dfrac{\underline{r}}{r^2}\right) = \dfrac{1}{r^2}\ (r \neq 0)$ where $\underline{r}$ carries the usual meaning.

6. **(a)** State Stokes' theorem, stating the meanings of any symbols used.

(b) Verify Stokes' theorem for the vector field $\underline{F} = -y^3\underline{i} + x^3\underline{j}$ for the case of a circle of radius R with centre at the origin.

(c) Prove that vector field $\underline{A} = (6xy + z^3)\underline{i} + (3x^2 - z)\underline{j} + (3xz^2 - y)\underline{k}$ is irrotational and find a scalar function such that $\underline{A} = \underline{\nabla} f$.