

The Open University of Sri Lanka
 B.Sc./B.Ed Degree Programme – Level 04
 Final Examination 2009/2010
 Pure Mathematics
 PMU 2191/PME 4191 – Vector Analysis



Duration :- Two and Half (2 ½) Hours.

Date :- 05-01-2010.

Time:- 1.00 p.m. – 3.30 p.m.

Answer Four Questions Only.

1. (a) If $\sin u = \frac{x^2 + y^2}{x + y}$, show that

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ and

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u.$



(b) Using linear approximation obtain an approximate value for the real number, $7.96\sqrt{9.99 - (1.01)^2}.$

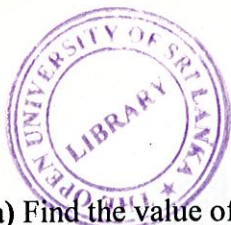
(c) Find the Taylor's expansion of $f(x, y) = e^y \ln(1+x)$ about the point (0, 0), up to 2nd order terms.

2. (a) (i) Define a stationary point of a single valued function $f(x, y)$, defined over the domain D . Explain briefly how you would determine its nature.

(ii) Find the maximum and minimum values of the function $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ and determine their nature.

(b) It is given that the directional derivative of $f(x, y, z) = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has a maximum of magnitude 64 in a direction parallel to z-axis. Show that $a = 6$, $b = 24$ and $c = -8$.

(c) The surface $ax^2 - byz = (a+2)x$ is orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, -1, 2). Find the constants a and b .



3. (a) Find the value of the surface integral of the function $f(x, y) = y^2/x^2$ over the region bounded by $y = x$, $x = 1$ and $y = 2$.

(b) Evaluate $\int_S e^{x^2+y^2} dA$, where S is finite region in the 1st quadrant bounded by $x^2 + y^2 = a^2$, $y = 0$, and $x = 0$.

(c) Find the volume integral of the function $f(x, y, z) = x^2 + y^2$ over the region bounded by the surfaces $z = x^2 + y^2$, $x = 0$, $y = 0$ and $z = 1$.

4. Let $U = x^2 + y^2 + z^2$ be a scalar field and S be the finite (closed) surface of the cylinder bounded by $x^2 + y^2 = a^2$, $z = 0$, and $z = h$. Compute Surface integral $\int_S U \underline{n} dS$ over the cylinder where \underline{n} is a unit vector normal to the surface S .

Hence verify that $\int_S U \underline{n} dS = \int_V \text{grad } U dV$ where V is the finite volume enclosed by S .

5. (a) State Gauss' divergence theorem, stating the meanings of any symbols used..

(b) Verify Gauss' divergence theorem for the vector field $\underline{F} = z^3 \underline{k}$, considering the region enclosed by the surface S of a sphere of radius R with centre at the origin.

(c) Prove that (i) $\underline{\nabla} \cdot \underline{r} = 3$, (ii) $\underline{\nabla} \cdot \left(\frac{\underline{r}}{r^2} \right) = \frac{1}{r^2}$ ($r \neq 0$) where \underline{r} carries the usual meaning.

6. (a) State Stokes' theorem, stating the meanings of any symbols used.

(b) Verify Stokes' theorem for the vector field $\underline{F} = -y^3 \underline{i} + x^3 \underline{j}$ for the case of a circle of radius R with centre at the origin.

(c) Prove that vector field $\underline{A} = (6xy + z^3) \underline{i} + (3x^2 - z) \underline{j} + (3xz^2 - y) \underline{k}$ is irrotational and find a scalar function such that $\underline{A} = \underline{\nabla} f$.