The Open University of Sri Lanka
B.Sc /B.Ed. Degree Programme/ Continuing Education Programme
Final Examination-2009/2010
AMU 2183/AME 4183-Mathematical Modeling III
Level 04-Applied Mathematics

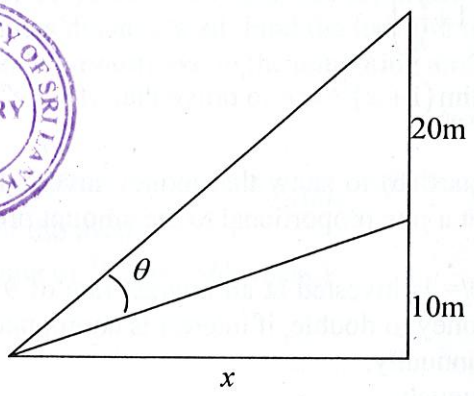
Duration:Two and half hours

Date:13-01-2010 **Tme:9.30 a.m-12.00 noon**

Answer four questions only.

01. (a) Find the volume V of the largest right circular cone that can be inscribed in a sphere of radius r.

(b) A movie screen on a wall is 20m high and 10m above the floor. At what distance x from the front of the room should you position yourself so that the viewing angle θ of the movie screen is as large as possible? Neglect the height of the viewer.



02. A uniform light beam PQ of length a is simply suspended in a horizontal position at its both ends and a concentrated load W is kept at distance b ($b < a/2$) from the end P.
Show that the maximum deflection of the beam is given by,

$$\frac{W(a-b)[b(2a-b)]^{3/2}}{9\sqrt{3}Ka}.$$

Also, show that the maximum deflection occurs at a distance $\left\{\frac{b(2a-b)}{3}\right\}^{1/2}$ from the end P, where K is the flexural rigidity of the beam.

03. Suppose Rs. P is invested at an annual interest rate of $r\times 100\%$. If the accumulated interest is credited to the account at the end of the year, the interest is said to be compounded annually; if it is credited at the end of each 6-month period, then it is said to be compounded semiannually; and if it is credited at the end of each 3-month period, then it is said to be compounded quarterly. The more frequently the interest is compounded, the better it is for the investor since more of the interest is itself earning interest.

(a) Show that if interest is compounded n times a year equally spaced intervals, then the value A of the investment after t years is

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

(b) One can imagine interest to be compounded each day, each hour, each time, and so forth. Carried to the limit one can conceive of interest compounded at each instant of time; this is called continuous compounding. Thus, from part (a), the value A of Rs. P after t years when invested at an annual rate $r\times 100\%$, compounded continuously, is

$$A = \lim_{n\to\infty} P\left(1+\frac{r}{n}\right)^{nt}$$

Use the fact that $\lim_{x\to 0}(1+x)^{1/x} = e$ to prove that $A = Pe^{rt}$.

(c) Use the result in part (b) to show that money invested at continuous compound interest increases at a rate proportional to the amount present.

(d) A sum of Rs.5000/= is invested at an interest rate of 9% per year. Find the time required for the money to double, if interest is compounded:
(i) Semi- annually,
(ii) Continuously.

(e) Find the effective rate for an investment that earns 5.5% per year, compounded continuously.

04. A college hostel accommodates 100 students, each of whom is susceptible to a certain virus infection. A simple model of epidemics assumes that during the course of an epidemic, the rate of change with respect to time of the number of infected students $I(t)$ is proportional to the number of uninfected students.

(a) If at time t=0, a single student becomes infected. Show that the number of infected students at any time t, $I(t)$ is given by

$$I(t) = \frac{100e^{100kt}}{99+e^{100kt}}.$$

(b) If k=0.01, when t is measured in days, find the value of the rate of new cases $I_1(t)$ at the end of each day for the first two days.

(c) Find the time it would take for half of the students in the hostel to get infected.

05. Suppose a school is holding a "family friendly" event and students have been pre-selling tickets to the event. An adult ticket costs Rs 50/= more than a child ticket. Suppose it is required to estimate the expected ticket revenue separately from child tickets and from adult tickets.
However, according to the ticket seller no record was available on the number of adult tickets and the number of child tickets sold separately. Only information known is that 199 tickets had been sold in total for Rs 23,850/=.

(i) Find the revenues from child tickets and adult tickets.

(ii) In the last year, 145 had attended the event of whom 84 were adults. In comparison to the last year, find the percentage decrease or increase in the number of adult tickets this year.

06. (a) A breeder reactor converts the relatively stable Uranium 238 into the isotope Plutonium 239. After 15 years, it is found that 0.043 per cent of the initial amount A_0 of the Plutonium has disintegrated. Find the half life time of this isotope, if the rate of disintegration is proportional to the remaining amount.

(b) A fossilized bone is found to contain $1/1000$ of the original amount of ^{14}C. Determine the age of the fossil.
(Take the half life time of ^{14}C as 5568 years.)