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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme
FINAL EXAMINATION-2009/2010
AMU 2182/ AME 4182- STATISTICS I
APPLIED MATHEMATICS – LEVEL 04

Duration: Two and Half Hours.

Date: 29.12.2009	**Time: 9.30 a.m.- 12.00 noon**

Answer FOUR questions only.

(1)

(a) State whether the following statements are true or false. Justify your answers.

(i) If $P(A \cup B) < P(A) + P(B)$ then, the two events cannot be mutually exclusive.

(ii) If ***A*** and ***B*** are mutually exclusive then ***A*** and ***B*** are not independent.

(iii) If ***A*** and ***B*** are independent, then they are mutually exclusive.

(b) Given that $P(A) = 1/4$, $P(B) = 1/3$ and $P(A' \cap B) = 1/3$ find

(i) the relationship between ***A*** and ***B***.

(ii) the value of $P(A/B)$

(iii) the value of $P(A' \cap B')$

(c) Three events ***A, B, C*** are defined on the sample space. The events ***A*** and ***C*** are mutually exclusive. The events ***A*** and ***B*** are independent.

Given that
$P(A) = 1/3$, $P(C) = 1/6$ and $P(A\ U\ B) = 2/3$
find

(i) $P(A \cup C)$

(ii) $P(B)$

(iii) $P(A \cap B)$

(d) Two groups of candidates are competing for the position of the board of directors of a company. The probabilities that the candidates group ***A*** and group ***B*** will win are o.6 and 0.4 respectively. If ***A*** wins the probability of introducing a new product is 0.8 and the corresponding probability if ***B*** wins is 0.3. What is the probability that new product will be introduced?

(2)

A company that produces a certain pain killer claims that the time X (in minutes) to relieve a pain after a tablet of their pain killer has the cumulative distribution function

$$F_X(x) = 1 - e^{-kx}; \ x > 0, \ k > 0$$

Median time to relieve a pain after taking a tablet is five minutes.

(a) Calculate the value k.
(b) Derive the density function of X.
(c) Show that the moment generating function of X is

$$M_X(t) = \left(1 - \frac{t}{k}\right)^{-1}; \ -k < t < k$$

Hence find the mean time to relieve a pain after taking a tablet.

(d) What is the probability that a randomly selected patient with a pain will get relief within 7 minutes after taking a tablet?

(e) Suppose 75% of patients with pain get relief within t minutes after taking a tablet of their pain killer. Find the value of t.

(3)

Front tire of a particular car is supposed to be filled to a pressure of 26 p.s.i. Suppose the actual pressure in the right tire is a random variable denoted by X and the actual air pressure in the left tire is a random variable denoted by Y.

The joint density of X and Y has the probability density function given below.

$$f_{X,Y}(x,y) = \begin{cases} k(x^2 + y^2); & 20 \le x \le 30, \ 20 \le y \le 30 \\ 0; & \text{otherwise} \end{cases}$$

(a) Find the value of k.
(b) What is the probability that both tires are filled at least to the required pressure?
(c) What is the probability that at least one tire is under filled?
(d) What is the probability that both tires are under filled given that the right tire is under filled?

(4)

(a) Explain the distinctive features of Binomial, Poisson and Normal distributions. Under what conditions the normal distribution can be used to approximate the binomial probabilities?

(b) A student is using a binomial distribution to model the following situation. State any assumptions that must be made and give possible values for the parameters n and p.

Some traffic lights have three phases: ***stop 45%*** of the time, ***wait or get ready 10%*** of the time and ***go 45%*** of the time. Assuming that you only cross a traffic light when it is in the ***go*** position model the number of times that you have to ***wait or stop*** on your way to school given that there are 8 sets of traffic lights.
Hence find the mean number of times that you have to ***wait or stop*** on your way to school.

(c) The number of admissions to an emergency ward of a hospital on a Saturday morning during the period beginning at 12.00 midnight and ending at 2.00 a.m. is found to have a Poisson distribution with an average of 3.5 admissions. During this period of a particular Saturday morning,

(i) What is the probability that none will be admitted?

(ii) What is the probability that two to six persons (inclusive) will be admitted?

(d) In a large lot of radios produced by a company, it is known that 10 percent of them are defective. A certain retailer purchase 200 of those radios from this company. What is the probability that more than 30 of them are defective?

(5)

(a) What is a normal distribution?
Highlight its important properties.
Distinguish between normal distribution and standard normal distribution.

(b) The lifetime of an automotive battery is normally distributed with mean 38 months and standard deviation 9 months.

(i) Find the probability that lifetime of a randomly selected battery is in between 29 months and 47 months.

(ii) Find the probability that lifetime of a randomly selected battery exceeds 20 months.

(iii) In setting warranties on batteries, manufacturers want to set the time limit in such way that few batteries will have to be refilled at the manufactures expense. However they want the buyer to have some degree of protection against manufacturing over a period of time after the purchase. The manufacturer would like to set the expiration time of the warranty at such a level that 95% batteries made will remain in working condition throughout the warranty period. What should be the warranty period?

6)

(a) Describe in your own words the central limit theorem. Why is it such an important theorem in statistics?

(b) A random sample of 3 beetles is drawn from a population of beetles whose length **X** is normally distributed with mean 2.4cm and standard deviation of 0.36 cm. the mean length $\overline{X}$, is calculated.

(i.) State the distribution of $\overline{X}$, giving the values of its parameters.

(ii.) Find $P(\overline{X} > 2.5)$

(iii.) State which of the answers for the above parts, if any, depend on the central limit theorem.

(c) A machine is set to produce ball-bearings with mean diameter 1.2cm. Each day a random samp of 50 ball-bearings are selected and the diameters are accurately measured. If the sample mea diameter lies outside the range 1.18cm to 1.22cm then it will be taken as evidence that mea diameter of the ball bearing produced is not 1.2 cm. The machine will then be stopped ar adjustments made to it. Assuming that the diameter has a standard deviation of 0.075 cm, find th probability of,

(i) The machine being stopped unnecessarily

(ii) The machine not being stopped when the mean diameter of the ball- bearing product is 1.15cm.

(iii) It is given that Pr(sample mean diameter$<a$) = 0.5, find the value of a.

(iv) Do your answers for (i), (ii), (iii) depend on the central limit theorem? Justify your answer.