The Open University of Sri Lanka
B.Sc Degree Programme/Continuing Education Programme
Final examination -2009/2010
AMU 2181/AME4181-Mathematical Modeling I
Level 04-Applied Mathematics
Duration:- Two and Half Hours.

Date:-21/01/2010 **Time:-9.30 a.m.-12.00 noon.**

Answer Four Questions.

(01)(i) What are the essential characteristics of a linear programming model?

(ii) The ABC Electric Appliance Company produces two products: refrigerators and ranges. Production takes place in two separate departments. Refrigerators are produced in department I and ranges are produced in department II. The company's two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 refrigerators in department I and 35 ranges in department II, because of limited available facilities in the two departments. The company regularly employs a total of 60 workers in the two depts. A refrigerator requires 2 man-weeks of labour, while a range requires 1 man-week of labour. A refrigerator contributes a profit of Rs. 60 and a range contributes a profit of Rs.40. Formulate the problem as a linear programming problem. How many units of refrigerators and ranges should be the company produce to realize a maximum profit?

(02) (i) Solve the following problem by simplex method.

$$\begin{aligned} \text{Max } Z &= 2x_1 + 4x_2 + x_3 + x_4 \\ \text{subject to } & x_1 + 3x_2 + x_4 \le 4 \\ & 2x_1 + x_2 \le 3 \\ & x_2 + 4x_3 + x_4 \le 3 \\ & x_i \ge 0, (i = 1,2,3,4) \end{aligned}$$

(ii) Show that the following problem has an unbounded optimal solution.

$$\begin{aligned} \text{Max } Z &= 2x_1 + x_2 \\ \text{subject to } & x_1 - x_2 \le 10 \\ & 2x_1 - x_2 \le 40 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(03) Solve the following problem using Big- M method.

$$\text{Minimize } Z = 4x_1 + x_2$$
$$\text{subject to } 3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 3x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

Verify the solution using the graphical approach.

(04) Consider the following primal problem:

$$\text{Minimize } Z = 3x_1 + 15x_2 + 5x_3 + 6x_4$$
$$\text{Subject to } x_1 + 6x_2 + 3x_3 + x_4 \geq 2$$
$$-2x_1 + 5x_2 - x_3 + 3x_4 \geq -3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

(i) Give the dual linear problem and hence solve.

(ii) Obtain the optimal solution to the primal problem using the optimal solution to the dual problem.

(05) Consider the following problem.

$$\text{Maximize } Z = 6x_1 + 8x_2$$
Subject to
$$5x_1 + 10x_2 \leq 60$$
$$4x_1 + 4x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

The optimum simplex table for the above problem is given below:

Basic variable	X_1	X_2	S_1	S_2	solution
X_2	0	1	1/5	-1/4	2
X_1	1	0	-1/5	1/2	8
-Z	0	0	-2/5	-1	-64

Hence, write the optimum solution to the problem,

(i) if the right-hand side constants of constraint 1 and constraint 2 change from 60 and 40 to 40 and 20, respectively.

(ii) if the right-hand side constants of the constraints change from 60 and 40 to 20 and 40 respectively.

(06) Consider the following problem:

$$\text{Maximize } Z = 10x_1 + 15x_2 + 20x_3$$
$$2x_1 + 4x_2 + 6x_3 \leq 24$$
$$3x_1 + 9x_2 + 6x_3 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

The optimum table of the above problem is given below.

Basic variable	X_1	X_2	X_3	S_1	S_2	solution
X_3	0	-1	1	1/2	-1/3	2
X_1	1	5	0	-1	1	6
-Z	0	-15	0	0	-10/3	-100

(i) Find the range of the objective function coefficient C_1 of the variable x_1 such that the optimality is unaffected.

(ii) Find the range of the objective function coefficient C_2 of the variable x_2 such that the optimality is unaffected.

(iii) Check whether the optimality is affected, if the profit coefficients change from $(10,15,20)$ to $(7,14,15)$. If so, find the revised optimum solution.