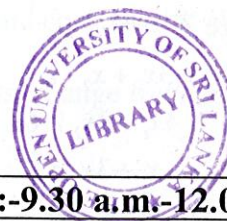


The Open University of Sri Lanka  
 B.Sc Degree Programme/Continuing Education Programme  
 Final examination -2009/2010  
 AMU 2181/AME4181-Mathematical Modeling I  
 Level 04-Applied Mathematics  
**Duration:- Two and Half Hours.**



**Date:-21/01/2010**

**Time:-9.30 a.m.-12.00 noon.**

**Answer Four Questions.**

(01)(i) What are the essential characteristics of a linear programming model?

(ii) The ABC Electric Appliance Company produces two products: refrigerators and ranges. Production takes place in two separate departments. Refrigerators are produced in department I and ranges are produced in department II. The company's two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 refrigerators in department I and 35 ranges in department II, because of limited available facilities in the two departments. The company regularly employs a total of 60 workers in the two depts. A refrigerator requires 2 man-weeks of labour, while a range requires 1 man-week of labour. A refrigerator contributes a profit of Rs. 60 and a range contributes a profit of Rs.40. Formulate the problem as a linear programming problem. How many units of refrigerators and ranges should be the company produce to realize a maximum profit?

(02) (i) Solve the following problem by simplex method.

$$\begin{aligned} \text{Max } Z &= 2x_1 + 4x_2 + x_3 + x_4 \\ \text{subject to } x_1 + 3x_2 + x_4 &\leq 4 \\ 2x_1 + x_2 &\leq 3 \\ x_2 + 4x_3 + x_4 &\leq 3 \\ x_i &\geq 0, (i = 1, 2, 3, 4) \end{aligned}$$

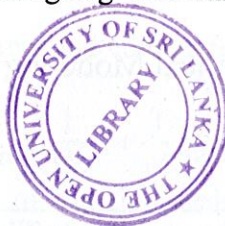
(ii) Show that the following problem has an unbounded optimal solution.

$$\begin{aligned} \text{Max } Z &= 2x_1 + x_2 \\ \text{subject to } x_1 - x_2 &\leq 10 \\ 2x_1 - x_2 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$



(03) Solve the following problem using Big- M method.

$$\begin{aligned} \text{Minimize } Z &= 4x_1 + x_2 \\ \text{subject to } 3x_1 + x_2 &= 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 3x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$



Verify the solution using the graphical approach.

(04) Consider the following primal problem:

$$\begin{aligned} \text{Minimize } Z &= 3x_1 + 15x_2 + 5x_3 + 6x_4 \\ \text{Subject to } x_1 + 6x_2 + 3x_3 + x_4 &\geq 2 \\ -2x_1 + 5x_2 - x_3 + 3x_4 &\geq -3 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

- (i) Give the dual linear problem and hence solve.
- (ii) Obtain the optimal solution to the primal problem using the optimal solution to the dual problem.

(05) Consider the following problem.

$$\begin{aligned} \text{Maximize } Z &= 6x_1 + 8x_2 \\ \text{Subject to } 5x_1 + 10x_2 &\leq 60 \\ 4x_1 + 4x_2 &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

The optimum simplex table for the above problem is given below:

Basic variable	$X_1$	$X_2$	$S_1$	$S_2$	solution
$X_2$	0	1	$1/5$	$-1/4$	2
$X_1$	1	0	$-1/5$	$1/2$	8
$-Z$	0	0	$-2/5$	-1	-64

Hence, write the optimum solution to the problem,

- (i) if the right-hand side constants of constraint 1 and constraint 2 change from 60 and 40 to 40 and 20, respectively.
- (ii) if the right-hand side constants of the constraints change from 60 and 40 to 20 and 40 respectively.

(06) Consider the following problem:

$$\text{Maximize } Z = 10x_1 + 15x_2 + 20x_3$$

$$2x_1 + 4x_2 + 6x_3 \leq 24$$

$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$



The optimum table of the above problem is given below.

Basic variable	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	solution
$X_3$	0	-1	1	1/2	-1/3	2
$X_1$	1	5	0	-1	1	6
-Z	0	-15	0	0	-10/3	-100

- (i) Find the range of the objective function coefficient  $C_1$  of the variable  $x_1$  such that the optimality is unaffected.
- (ii) Find the range of the objective function coefficient  $C_2$  of the variable  $x_2$  such that the optimality is unaffected.
- (iii) Check whether the optimality is affected, if the profit coefficients change from (10,15,20) to (7,14,15). If so, find the revised optimum solution.