The Open University of Sri Lanka
Foundation Course in Science – Level 02
Open Book Test (OBT) 2004/2005
MAF 2301/MAE 2301 - Pure Mathematics

Duration :- One and Half Hours.

Date :- 10-01-2006 **Time:- 1.30 p.m. – 3.00 p.m.**

Answer All Questions.

01. (a) Prove that $\frac{d}{dx}\left(\ln\left|x+\sqrt{x^2-a^2}\right|\right)=\frac{1}{\sqrt{x^2-a^2}}$.

Hence show that $\int\frac{1}{\sqrt{x^2-a^2}}dx=\ln\left|x+\sqrt{x^2-a^2}\right|$.

Hence evaluate $\int\frac{1}{\sqrt{x^2+x-1}}dx$.

(b) If $I_n=\int_0^{\pi/2}\cos^n x\,dx$, prove that $I_n=\left(\frac{n-1}{n}\right)I_{n-2};\ n\geq 2$.

Hence find the value of I_5.

02. (a) Evaluate,

(i) $\int_0^1\left(x^2+1\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2dx.$

(ii) $\int_0^{\pi/4}\sin 3x(1+\cos 2x)dx.$

(iii) $\int_0^{3/4}3x\sqrt{1+x^2}\,dx.$

(b) Find the area enclosed between the curve $y=x^3$ and the straight line $y=x$.

03 (a) Prove by Induction $\frac{1}{4.1^2-1}+\frac{1}{4.2^2-1}+\frac{1}{4.3^2-1}+\ldots+\frac{1}{4.n^2-1}=\frac{1}{2}-\frac{1}{4n+2}$.

(b) Hence find the sum $\sum_{r=1}^{\infty}\frac{1}{(2r-1)(2r+1)}$.