The Open University of Sri Lanka

B.Sc/B.Ed Degree programme

Pure Mathematics – Level 03

PEU3300 – Mathematical Logic & Mathematical Proofs

Final Examination - 2024/2025

Date: 05.12.2024	**Time: 09.30 a.m. – 11.30 a.m.**

General Instructions

- **This paper consists of TWO sections, Section A and Section B. Section A is compulsory and it consists of THREE Structured Essay Questions and carries 100 marks.**
- **Section B consists of FIVE essay type questions and answer only THREE of them.**
- **Each question in Section B carries 100 marks.**
- **This paper consists of 03 pages.**

SECTION A

1. Answer All Questions.

(a) Write an equivalent statement for each of the following statements:

(i) If x is prime, then $\sqrt{x}$ is not a rational number.

(ii) x is a non-zero rational number if and only if $\tan(x)$ is not a rational number.

(b) Consider the statement, $P(x, y)$: $x + y$ is divisible by 3, where the domain of x is $A = \{2, 4, 5, 6\}$ and the domain of y is $B = \{4, 7, 13\}$.

Determine the truth values of the following statements by providing justifications.

(i) For each $x \in A$, for each $y \in B$ such that $P(x, y)$.

(ii) There exists a $x \in A$ such that for each $y \in B$, $P(x, y)$.

(iii) There exists a $x \in A$, There exists a $y \in B$, such that $P(x, y)$.

(c) Consider the statement "If $6n + 1$ is odd then n is even."

If the proof uses 'Proof by Contraposition,' write down the assumptions you make.

SECTION B

Answer THREE questions ONLY.

2. (a) Prove that product of odd integer and an even integer is an even integer.

(b) Consider the statement "If $5n + 2$ is an even integer then n is an even integer.

Prove the above statement using each of the following methods:

(i) Method of Conditional proof

(ii) Proof by Contraposition

(c) Prove the statement '$2 + \sqrt{3}$ is irrational' using the method of Proof by Contradiction.

3. Prove or disprove each of the following statements.

(a) For each $n \in \mathbb{N}$, if $5n + 3$ is prime then $7n + 1$ is prime.

(b) There exists a rational number x, such that $5x^2 - 20 = 0$.

(c) $|x| = |y|$ if and only if $x = y$, where $(x, y) \in \{(1,2), (2, -2), (6,6)\}$.

(d) If n is an even integer, then $3n^3$ is an even integer.

(e) For each $n \in \mathbb{Z}$, $4 \mid (n^2 + 2)$.

4. (a) Let p and q be statements. Write an equivalent conjunctive statement for the bi-conditional statement $p \Leftrightarrow q$.

(b) Consider the statement, for $n \in \mathbb{Z}, 3n^3 + 4n^2 + 5$ is odd if and only if n is even.

(i) The proof of the above statement is divided into two parts. Below is first part of the proof. Fill in the blanks to complete it.

.....(1)

Suppose n is odd. Then $n = 2m + 1$ for some $m \in \mathbb{Z}$.

.....(2)

Since $12m^3 + 26m^2 + 17m + 6$ is an integer,

.....(3)

.....(4)

(ii) Prove the second part to complete the proof the statement.

(c) Let p and q be statements. Without using the truth tables, show that $p \Rightarrow (p \vee q)$ is a tautology.

5. Prove or disprove each of the following statements.

(a) For each $n \in \mathbb{Z}$, there exists a $m \in \mathbb{Z}$ such that $n = m - 5$.

(b) For each $n \in \mathbb{Z}$, for each $m \in \mathbb{Z}$, $n = m - 5$.

(c) There exists a $m \in \mathbb{Z}$ such that for each $n \in \mathbb{Z}, n = m - 5$.

(d) There exists a $n \in \mathbb{Z}$, there exists a $m \in \mathbb{Z}$ such that , $n = m - 5$.

6. (a) Let $x, y, z \in \mathbb{Z}$. Prove that if exactly two of the three integers x, y, z are even and the remaining one is odd, then $3x + 5y + 7z$ is odd.

(b) (i) Prove that $\frac{r}{\sqrt{2}}$ is irrational for any rational number r.

(ii) Using the result of part (i), prove that every non-zero rational number can be expressed as a product of two irrational numbers.

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