The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2006/2007
Pure Mathematics
PMU 3292/PME 5292 – Group Theory & Transformation – Paper II

Duration :- Two and Half Hours

Date :- 12-11-2006 **Time :- 1.00 p.m. – 3.30 p.m.**

Answer Four Questions Only.

01. In each of the following, find the left coset of H in G.

(a) $(G, +_4)$ is the group of integers modulo 4 and $H = \{0\}$.

(b) $(G, +_{15})$ is the group of integers modulo 15 and $H = \{0, 3, 6, 9, 12\}$.

(c) $(G, +_{15})$ is the group of integers modulo 15, $H = \{0, 5, 10\}$.

(d) $(\{1, -1, i, -i\}, \cdot)$ and $H = \{1, -1\}$.

02.(a) Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$ where e is the identity of G. Show that, for any $n \in N$, $m \in M$, $n \cdot m = m \cdot n$.

(b) If N is a normal subgroup of order 2 of a group G, then show that $N \subseteq Z(G)$, the center of G.

03. (a) Let H, K be normal subgroups of G. Prove that HK is a normal subgroup of G.

(b) Prove that a subgroup H of group G is a normal subgroup of G if and only if

$$xy \in H \Rightarrow yx \in H \quad \forall\, x, y \in G.$$

04. Consider the set G consisting of four permutations;

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

Show that (G, o) is an abelian group.

05.(a) Express each of the following permutation as a product of disjoint cycles:

(i) (1 2 3)(1 2)

(ii) (1 3 2 4)(1 2 3)

(iii) (1 2 3)(4 5)(1 4 2 5 3)

(iv) (1 2 3)(4 5)(1 5 3 4 2)

(b) Compute the product $T^{-1}ST$ for the following permutations T and S:

(i) $T = (1\ \ 2)$, $S = (1\ \ 2\ \ 3)$

(ii) $T = (1\ \ 3)(2\ \ 4)$, $S = (1\ \ 2\ \ 3\ \ 4)$

(iii) $T = (1\ \ 2\ \ 3)$, $S = (1\ \ 2\ \ 3\ \ 4)$.

06. Let $G = Z \times Z$ be a set and o be a binary operation on G defined by

$(a, b) \text{ o } (c, d) = (a + c, b + d)$.

(a) Show that (G, o) is a group. What is the identity element of G.

(b) Show that the mapping $f : G \to Z$ given by $f(a, b) = a$ is a homomorphism from (G, o) onto $(Z, +)$. What is kernel of f?

(c) $H = \{(a, a) \mid a \in Z\}$. Prove that (H, o) is a subgroup of (G, o), and isomorphic to $(Z, +)$ under the function f.