THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. Degree Programme

OPEN ELECTIVE- LEVEL 05

ADU5319- DESIGN AND ANALYSIS OF EXPERIMENTS

FINAL EXAMINATION 2023/2024

Duration: Two Hours

Date: 23.03.2024 **Time: 01.30p.m- 03.30p.m**

Instructions:

- This question paper consists of 06 questions and 05 pages.
- This question paper consists of two parts: Part *A* and Part *B*. **Part *A* is compulsory.** Part *B* consists of five (05) essay-type questions of which **three (03) are to be answered.**
- Statistical tables are attached at the end.
- Non-programmable calculators are permitted.
- Consider the **level of significance as 0.05** for all the tests.

PART *A* *(Question 01 is compulsory)*

Question 01

An experiment was conducted to compare the durability of 5 green fabric dyes, say D_1, D_2, D_3, D_4, and D_5, for washing. A sample of 30 plain white t-shirts was obtained and randomized so that 6 received each dye (with each shirt receiving exactly one dye). A measure of the color brightness of the shirts after 10 wash/dry cycles is obtained (with higher scores representing brighter color). The error sum of squares is (SSE) 2000. The mean scores for the 5 dyes are:

$$\bar{Y}_{D1} = 30,\ \bar{Y}_{D2} = 25,\ \bar{Y}_{D3} = 40,\ \bar{Y}_{D4} = 35,\ \bar{Y}_{D5} = 20$$

(i) In relation to this study, write down the

a) Treatments

b) Experimental unit

c) Response variable

(ii) Write down the design used in this study. Justify your answer.

(iii) Specify the hypotheses to be tested.

(iv) Clearly describing the notations ad the assumptions, write down the mathematical model for the design you identified in part(ii).

(v) Calculate the Least Significant Difference (LSD) for comparing any two treatment means.

(vi) Using the LSD test statistic computed in part(v), test whether dyes 3 and 5 are significantly different. Clearly state your findings.

Part *B*

Question02

An industrial engineer wanted to conduct an experiment on eye focus time. He is interested in the effect of the distance of the object from the eye on the focus time. Four different distances are of interest. He has five subjects available for the experiment because there may be differences among individuals. The focus time data (in seconds) is given below:

Distance (ft)	**Subject**				
	1	**2**	**3**	**4**	**5**
4	10	6	6	6	6
6	7	6	6	1	6
8	5	3	3	2	5
10	6	4	4	2	3

Total uncorrected sum of squares = 555

(i) Write down the design structure used in this experiment. Justify your answer.

(ii) Write down the hypotheses to be used.

(iii) Construct the analysis of variance table.

(iv) Test your hypotheses in part (ii) at 5% level of significance and write down your conclusions.

Question 03

An experiment was conducted to measure the effect of factory music on the productivity of workers. Four different music programs (*A, B, C, D*) were compared with no music (*E*). A single replication occupied the five working days of a week, with a different program being tested on each day. A design "*X*" is used to rotate the programs from week to week so that, over a five-week period, each program appeared once on any specific working day of the week.

The data, in percentages of target output, are as follows.

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Total
Week 1	E:82	A:123	B:118	D:114	C:108	**545**
Week 2	B:88	C:83	A:98	E:85	D:78	**432**
Week 3	D:69	E:81	C:96	B:102	A:87	**435**
Week 4	A:94	D:101	E:106	C:104	B:109	**514**
Week 5	C:80	B:109	D:90	A:100	E:85	**464**
Total	**413**	**497**	**508**	**505**	**467**	**2390**

Total Sum of Squares = 4586

Program	*A*	*B*	*C*	*D*	*E*
Total	502	526	471	452	439

(i) Name the design '*X*'. Give reasons for your answer.

(ii) Write down the assumptions you make when using this design.

(iii) Test whether all five music programs have the same effect on the productivity of workers.

(iv) Test the hypothesis that effects of music program *A* and *E* are equal. Clearly explain your answer.

Question 04

A measure of the yield of a certain industrial electrolytic process was observed for one week's operation of 30 electrolytic cells. Fifteen of the cells were operated using the standard control scheme (*A*) while eight of the others were operated under a modified control scheme (*B*) and the remaining seven were operated under a third control scheme (*C*). The results were as follows:

Control Scheme	Yields	Total
A	81.4 82.8 80.9 84.6 83.0 81.3 82.9 80.8 80.3 86.4 81.5 79.2 83.4 81.6 84.9	1235
B	92.8 95.3 94.8 96.0 91.2 88.9 91.9 85.4	736.3
C	85.0 80.3 81.0 79.5 82.1 80.9 81.5	570.3

Total Sum of Squares= 747.428

(i) Making the usual assumptions for an analysis of variance, test whether there is evidence in the results to suggest that there are differences between the mean yields given by these three control schemes.

(ii) Clearly describing the notations, write down contrasts for the following comparisons.
   a) standard scheme (*A*) with the new schemes (*B, C*)
   b) Schemes *B* with *C*

(iii) Are the contrasts in (a) and (b) stated in part (ii) orthogonal comparisons? Justify your answer.

Question 05

It is needed to study the effect of the brand and the temperature at the location where it is utilized on the lifetime of a generator. For that, an engineer has decided to test two brands (*A* and *B*) of generators at two temperature levels (15°C, 25°C). In the experiment, four groups of generators are randomly selected such that four machines are observed in each combination of brand of the generator and temperature level. The collected data are as follows:

Brand	**Temperature level**	
	15°C	**25°C**
A	88 145	51 58
	152 175	37 65
B	170 122	152 102
	174 156	162 148

Total Sum of Squares = 34743.44

(i) Write down the treatment structure of the experiment. Justify your answer.

(ii) Write down two advantages of this treatment structure.

(iii)Test whether the effects of brands of generators and temperature on the lifetime of generators are significant or not. Clearly explain your answer.

Question 06

Two varieties of wheat (*V*) are grown in two different fertility regimes (*F*). The field was divided into two blocks with four whole plots. Each of the four fertilizer levels was randomly assigned to one whole plot within a block. Each whole plot was divided into two subplots, and each variety of wheat was randomly assigned to one subplot within each whole plot. The results are accompanied in the following table.

		Block 1	
		Variety	
		V1	*V2*
Fertilizer	*F1*	36.4	38.9
	F2	36.7	38.2
	F3	34.8	36.4
	F4	40.5	41.0

		Block 2	
		Variety	
		V1	*V2*
Fertilizer	*F1*	42.6	41.3
	F2	42.7	41.6
	F3	43.6	42.8
	F4	45.5	48.6

The following summary statistics were computed by whole plot and sub plot.

Total Sum of Squares (SS Total)	204.45
Total Sum of Squares of whole plot (SS Total WP)	192.22
Total Sum of Squares of sub plot(SS Total SP)	57.99
Block Sum of Square (SS Block)	131.1025

(i) Write down the treatment and design structures for the whole plot and sub plot parts of the experiment.

(ii) Carry out an ANOVA and test the significance of the effects at 5% level.

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Appendix 4a

5 per cent Points of the *F*-distribution

Column represents degrees of freedom (ν_1) for numerator of *F*-test
Row represents degrees of freedom (ν_2) for denominator of *F*-test

	1	2	3	4	5	6	7	8	9	10	12	24	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.785	8.745	8.638	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.788	2.609	2.405
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177	2.104	1.901	1.638

(continued)

Critical values of *t* for two-tailed tests

Significance level (α)

Degrees of freedom *(df)*	0.2	0.15	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	4.165	6.314	12.706	25.452	63.657	127.321	636.619	1273.239
2	1.886	2.282	2.920	4.303	6.205	9.925	14.089	31.599	44.705
3	1.638	1.924	2.353	3.182	4.177	5.841	7.453	12.924	16.326
4	1.533	1.778	2.132	2.776	3.495	4.604	5.598	8.610	10.306
5	1.476	1.699	2.015	2.571	3.163	4.032	4.773	6.869	7.976
6	1.440	1.650	1.943	2.447	2.969	3.707	4.317	5.959	6.788
7	1.415	1.617	1.895	2.365	2.841	3.499	4.029	5.408	6.082
8	1.397	1.592	1.860	2.306	2.752	3.355	3.833	5.041	5.617
9	1.383	1.574	1.833	2.262	2.685	3.250	3.690	4.781	5.291
10	1.372	1.559	1.812	2.228	2.634	3.169	3.581	4.587	5.049
11	1.363	1.548	1.796	2.201	2.593	3.106	3.497	4.437	4.863
12	1.356	1.538	1.782	2.179	2.560	3.055	3.428	4.318	4.716
13	1.350	1.530	1.771	2.160	2.533	3.012	3.372	4.221	4.597
14	1.345	1.523	1.761	2.145	2.510	2.977	3.326	4.140	4.499
15	1.341	1.517	1.753	2.131	2.490	2.947	3.286	4.073	4.417
16	1.337	1.512	1.746	2.120	2.473	2.921	3.252	4.015	4.346
17	1.333	1.508	1.740	2.110	2.458	2.898	3.222	3.965	4.286
18	1.330	1.504	1.734	2.101	2.445	2.878	3.197	3.922	4.233
19	1.328	1.500	1.729	2.093	2.433	2.861	3.174	3.883	4.187
20	1.325	1.497	1.725	2.086	2.423	2.845	3.153	3.850	4.146
21	1.323	1.494	1.721	2.080	2.414	2.831	3.135	3.819	4.110
22	1.321	1.492	1.717	2.074	2.405	2.819	3.119	3.792	4.077
23	1.319	1.489	1.714	2.069	2.398	2.807	3.104	3.768	4.047
24	1.318	1.487	1.711	2.064	2.391	2.797	3.091	3.745	4.021
25	1.316	1.485	1.708	2.060	2.385	2.787	3.078	3.725	3.996
26	1.315	1.483	1.706	2.056	2.379	2.779	3.067	3.707	3.974
27	1.314	1.482	1.703	2.052	2.373	2.771	3.057	3.690	3.954
28	1.313	1.480	1.701	2.048	2.368	2.763	3.047	3.674	3.935
29	1.311	1.479	1.699	2.045	2.364	2.756	3.038	3.659	3.918
30	1.310	1.477	1.697	2.042	2.360	2.750	3.030	3.646	3.902
40	1.303	1.468	1.684	2.021	2.329	2.704	2.971	3.551	3.788
50	1.299	1.462	1.676	2.009	2.311	2.678	2.937	3.496	3.723
60	1.296	1.458	1.671	2.000	2.299	2.660	2.915	3.460	3.681
70	1.294	1.456	1.667	1.994	2.291	2.648	2.899	3.435	3.651
80	1.292	1.453	1.664	1.990	2.284	2.639	2.887	3.416	3.629
100	1.290	1.451	1.660	1.984	2.276	2.626	2.871	3.390	3.598
1000	1.282	1.441	1.646	1.962	2.245	2.581	2.813	3.300	3.492
Infinite	1.282	1.440	1.645	1.960	2.241	2.576	2.807	3.291	3.481