The Open University of Sri Lanka
B.Sc/ B.Ed. Degree Programme
Final Examination - 2023/2024
Applied Mathematics - Level 05
ADU5308 – Graph Theory

Duration: - Two hours

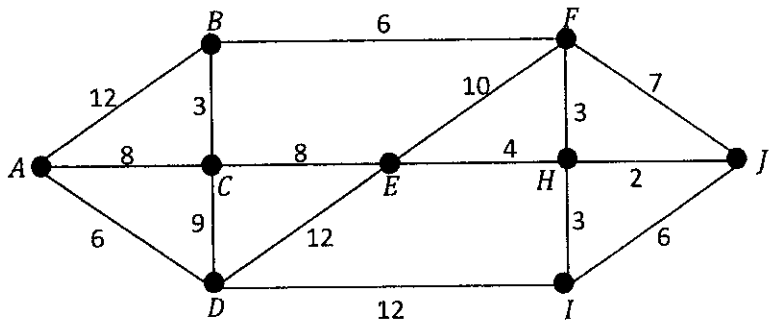
Date: - 02-04-2024 **Time: - 01.30 p.m. – 03.30 p.m.**

Answer **four** questions **only.**

1)

a) A regular graph of 9 vertices has total degree 72. Find the degree of each vertex and number of edges.

b) Find the shortest path from A to J of the weighted graph G.

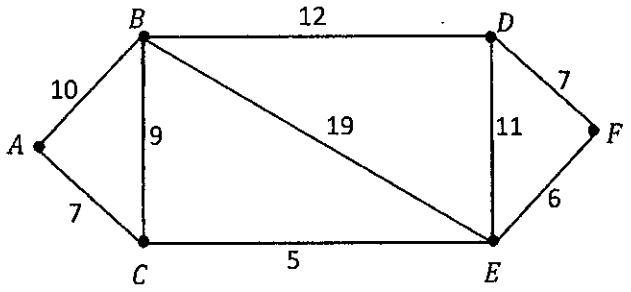


G

c) Define a **Spanning Tree** of a graph G.
Draw all the non-isomorphic spanning trees of the graph $K_4 - v$, where v is a vertex.

2)

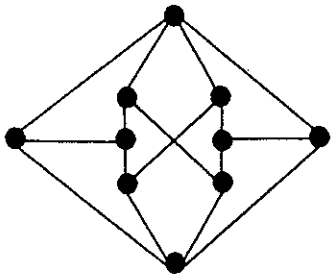
a) A postman collects his mail from one of the locations A, B, C, D, E, F deliver his mail and return to the starting location. Find the minimum distance he must traverse in order to deliver all the mail. Also, find at which locations he has to start his duties in order to complete the entire job in traveling minimum distance.



b) Draw the dual of the planar representation of K_4.

c) Define the **Chromatic Number** and **Chromatic Index** for a graph G.

i. Find the chromatic number and chromatic index of the given graph G.



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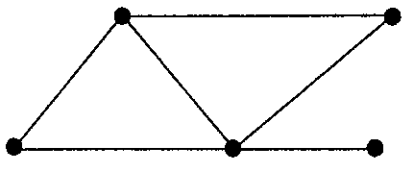
ii. Is the graph G satisfy the conditions of the **Four-color theorem** or not? Justify your answer.

3)

a) Consider the flowing graph G and construct the followings:



G

i. Subdivision graph $S(G)$.

ii. 2nd iterated line graph of G, $L^2(G)$.

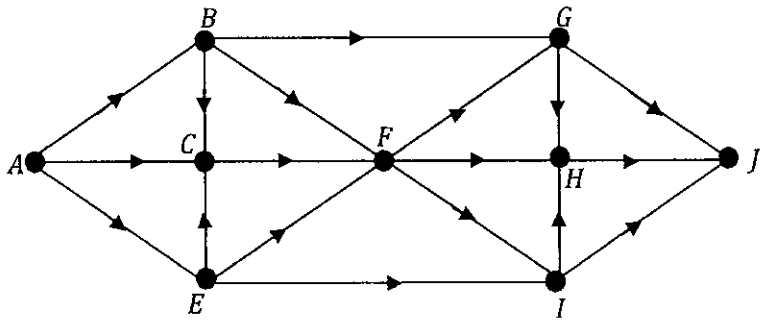
iii. Line graph $L_2(G)$.

b) Construct a connected graph G with points $p \geq 4$ such that G is not Eulerian but $L(G)$ is Eulerian.

c) Define a **Total Graph** $T(G)$.
Construct the total graph of C_3.

4)

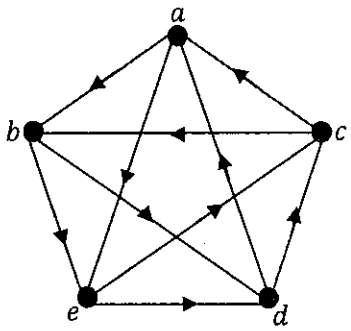
a) State the **Handshaking Dilemma**.
Consider the digraph $D = (V_1, A_1)$.



D

i. Find $d(A, a)$ for all $a \in V_1$.
ii. Find $d(a, J)$ for all $a \in V_1$.
iii. The digraph D is strong or not. Justify your answer.

b) In the following tournament, find



i. cycles of length 3, and 5,
ii. a closed Eulerian trail, and
iii. a Hamiltonian cycle.

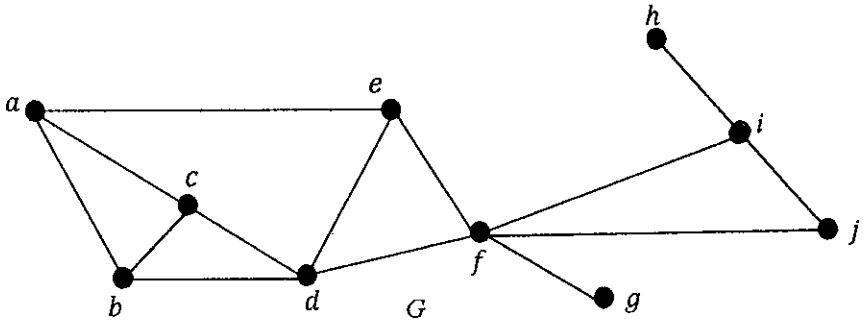
c) In the tournament given in part (b), verify that $\sum in\deg(v)^2 = \sum out\deg(v)^2$

5)

a) Define the terms **Cut point**, **Bridge**, and **Block** of a graph. Consider the following graph G :

i. Find all the cut point(s), bridge(s), and block(s) of the graph G.

ii. Construct the corresponding block graph and cut point graph for the graph G.



b) Define a **Complete Matching**.

Suppose that four girls w, x, y, z know six boys a, b, c, d, e, f as given in the following table.

Girl	Boys known by girl
w	a c f
x	b c d f
y	a d e
z	b e

i. Draw the bipartite graph corresponding to these relationships.

ii. Check the marriage condition for this problem.

6)

a) Consider the family $\mathfrak{I} = (S_1, S_2, S_3, S_4, S_5)$ of subset of $E = \{1, 2, 3, 4, 5\}$, where $S_1 = \{1, 2\}$, $S_2 = S_3 = \{2, 3, 4\}$, and $S_4 = S_5 = \{1, 3, 4, 5\}$.

i. Obtained the corresponding incident matrix.

ii. Find the term rank.

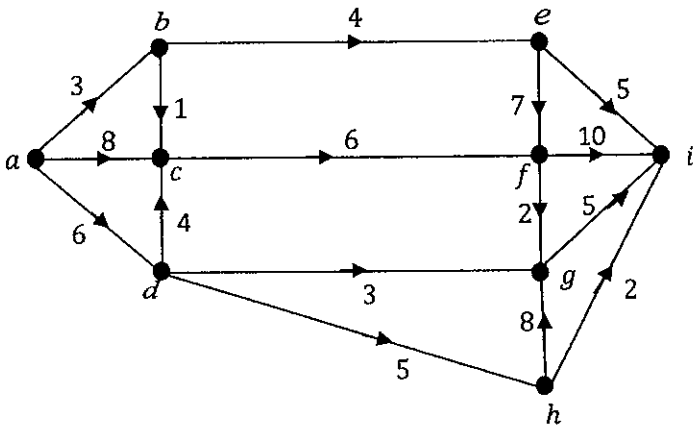
iii. State **Konig-Egervecy theorem** and verify it for this problem.

iv. Determine whether the above family has transversals or not. Justify your answer.

b) State the **Max-flow, Min-cut theorem.**

i. Verify Max-flow, min-cut theorem for the following Network N.

ii. Identify the saturated arcs for the Network N.



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