THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed DEGREE PROGRAMME-LEVEL 05
FINAL EXAMINATION-2023/2024
ADU5306 — FLUID MECHANICS
DURATION: TWO HOURS

Date: 09.04.2024 **Time: 09.30a.m. –11.30a.m.**

ANSWER FOUR QUESTIONS.

1. a) Derive the continuity equation of the form $\frac{D\rho}{Dt} + \rho\, div\left(\underline{q}\right) = 0$, for any arbitrary control volume of a moving fluid irrespective of its shape and size. $\underline{q}$ and ρ are fluid velocity and fluid density respectively.

   b) Fluid velocity at a point having spherical polar coordinates (r, θ, ω) has components given by $\underline{q} = \left[\frac{2\mu\cos\theta}{r^3}, \frac{\mu\sin\theta}{r^3}, 0\right]$ where μ is a constant. Show that this represents a possible motion of an incompressible fluid and find the equations of streamlines.

2. a) Derive the Euler's equation of motion for a perfect fluid of the form $\underline{F} - \frac{1}{\rho}\, grad\, p = \frac{D\underline{q}}{Dt}$.

   b) A stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to steady and diverging from the vertex of the cone, then prove that

   $\frac{v}{V} = \left(\frac{D^2}{d^2}\right) e^{(v^2 - V^2)/2K}$, where K is the pressure divided by the density and is a constant.

3. a) State Bernoulli's theorem for a fluid which is in steady motion without velocity potential and conservative field of force.

   b) A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area A is delivered atmospheric pressure at a place, where the sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from reservoir at a depth $\left(\frac{s^2}{2g}\right)\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$ below the pipe, s being the delivery per second.

4. a) State Kelvin's theorem.

b) Show that the function $\emptyset = U\left[r + \frac{a^2}{r}\right]\cos\theta - k\theta$, where the constants U, a, k are positive constants and (r, θ, z) denotes cylindrical polar coordinates, represents velocity potential of an irrotational motion of an incompressible fluid, by verifying that the velocity derived from it satisfies the equation of continuity. Find the velocity at infinity and the velocity at any point on the cylinder $r = a$. Show that there is a constant circulation in any circuit surrounding the cylinder.

5. a) State the continuity equation for incompressible fluid in terms of Cartesian Coordinates.

b) Let $q = \frac{-yi+xj}{x^2+y^2}$ be a velocity field for a fluid. Calculate the circulation around a square whose corners are at $(1,0), (2,0), (2,1), (1,1)$ considering the above q.

c) Determine streamlines for the flow whose velocity field is given by

$$u = \frac{x}{t+1}, v = \frac{y}{t+1}, w = \frac{z}{t+1}$$

6. A two-dimensional source of strength 2m is placed at point $A(a, 0)$ and a sink of strength m at the point $B(-a, 0)$.

a) Write down the complex potential,

b) Show that there is a point of stagnation at $C(-3a, 0)$.

c) Show that the speed q at any point P in the z-plane is $\frac{m(PC)}{PA.PB}$

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