The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
FINAL EXAMINATION 2023/2024
Level 05 - Applied Mathematics
ADU5305– Statistical Inference

Duration: - Two Hours.

DATE: - 07-04-2024 **Time: - 9.30 a.m. to 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.
Answer four questions only.

1.
Suppose weight of a certain product, produced by ABC Company, follows a normal distribution. However, the mean weight and variance weight of randomly selected product is unknown.

98.59	100.43	98.86	99.62	99.93	101.45	99.42	97.37
99.82	100.02	100.30	98.98	100.97	100.28	100.02	100.22

(i) What is the population of interest? Is the population finite? Justify your answer.

(ii) Derive the moment estimators for the mean and the variance of a randomly selected n products.

(iii) Weights of 16 randomly selected products in grams are given above. Using part (ii) estimate the mean and the variance of weight.

(iv) Estimate the standard error of the estimated mean given by you in part (iii).

(v) Using a suitable statistical test check the validity of the claim that the mean weight of the product is greater than 100 grams.

2.
(a) In an export process of mangoes by ABC Company, the sales manager is interested in the proportion θ of mangoes not up to the export quality in a batch purchased from an estate. Suppose batch contains 10000 mangoes. A random sample of 100 mangoes were tested and suppose that 2 of the mangoes were not according to the export quality.

(i) Construct a 95% confidence interval for $\boldsymbol{\theta}$. Do the calculations for three decimal places.

(ii) Construct 95% confidence interval for the total number of mangoes not up to the export quality. Hence comment on the claim that " total number of mangoes not up to the export quality in the batch can be 500". Justify the answer.

(b) The diameter of steel rods manufactured on an extraction machine is being investigated. A random sample of size n = 13 was selected from the production of the machine. The sample mean and the sample variance are $\bar{x} = 8.73\ mm$, $s^2 = 0.35\ mm^2$ respectively. Assume that the diameter of steel rods manufactured by extraction machine follows a normal distribution.

(i) Construct a 99% confidence interval for the mean of diameter of a randomly selected steel rod manufactured by the machine and interpret your answer.

(ii) Dose the data provide evidence to justify the claim " mean diameter of a randomly selected steel rod is greater than 10 mm" . Using a suitable statistical test write your answer. Use 5% level of significance.

3.

(a) Let $\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_n}$ be a random sample from a population with density given by $f(x; \boldsymbol{\theta})$. Let $\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2, \hat{\boldsymbol{\theta}}_3, \hat{\boldsymbol{\theta}}_4$ are functions of $\mathbf{X_1}, \mathbf{X_2}, \ldots \mathbf{X_n}$. Suppose $\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2\ \hat{\boldsymbol{\theta}}_3, \hat{\boldsymbol{\theta}}_4$ are unbiased estimators for the parameter $\boldsymbol{\theta}$ and $MSE(\ \hat{\theta}_3\ ) < MSE(\ \hat{\theta}_2\ )$. Prove or disprove each of the following statements:

(i) $Biase(\frac{\hat{\theta}_1+\hat{\theta}_3}{3} + \frac{\hat{\theta}_2+3\hat{\theta}_4}{12}) = 0$

(ii) $\boldsymbol{Var}(\hat{\boldsymbol{\theta}}_2) < Var(\hat{\boldsymbol{\theta}}_3)$ -

(b) Assignment marks and final examination marks of a particular subject for 15 students are given below:

Student Name	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Assignment mark	60	47	60	56	47	27	45	61	68	62	35	53	57	25	31
Final Mark	67	54	53	49	47	35	30	77	57	54	42	60	63	42	28

Using a suitable statistical test, test the validity of the claim that "Expected assignment mark is greater than the expected final examination mark for a randomly selected student". Use 5% level of significance.

4.

An investigation was conducted in to the dust content in the flue gases of two types of solid – fuel boilers. 13 boilers of type *A* and 9 boilers of type *B* were used under identical fueling and extraction conditions. Over a similar period, quantities, in grams, of dust deposited in similar traps inserted in each of the flues were recorded. Assume that these samples come from normal populations. Sample means of dust contents of type A and type B are 63.83 grams and 52.89 grams respectively. Sample standard deviations of the dust contents of type A and type B are 10.63 grams and 9.00 grams respectively.

(i) Test for an equality of population variances. Use 5% level of significance. Clearly state the findings.
(ii) Test for an equality of population means. Use 5% level of significance. Clearly state the findings.
(iii) Do the dust contents in the flue gases of two types of solid – fuel boilers have identical distribution? Justify your answer.
(iv) Construct 95% confidence interval for the differences of mean dust contents of type A and type B. comment on the results.

5.

(a) Briefly explain the following terms used in statistical hypothesis testing:

(i) Type I error and Type II error

(ii) Power of the test.

(b) Let $X_1, X_2, \ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{2-\theta} \quad ; \ 1 \le x \le 3-\theta ; \quad 0 < \theta < 1.$$

(i) Find the mean of the above distribution.

(ii) Derive the Moment estimator for mean of the above distribution.

(iii) Derive Maximum likelihood estimator for the mean of the above distribution.)

(iv) A random sample drawn from the above distribution is given in the following table:

2.39	1.90	2.16	1.67	2.15
1.05	1.92	1.82	2.15	1.92
2.43	1.59	1.58	2.07	1.38
1.19	1.35	2.38	1.90	1.74
1.33	1.98	2.14	1.88	1.12

(i) Estimate the mean of the above distribution using the moment estimators derived in part (i).
(ii) Estimate the mean of the above distribution using the maximum likelihood estimator derived in part(ii).

6.

(a) Suppose $\hat{\theta}$ is an estimator for the parameter θ. State whether the following statements are true or false. In each case justify your answer.

(i) $\hat{\theta}$ is an unbiased estimator for the parameter θ implies that $\hat{\theta}$ is a consistent estimator for the parameter θ

(ii) $Var(\hat{\theta}) = \frac{\theta}{n}$ and $\hat{\theta}$ is an unbiased estimator for parameter θ implies that $\hat{\theta}$ is a consistent estimator for parameter θ.

(b) Suppose weight of a certain product X, produced by ABC Company, follows normal distribution. However, the mean weight and variance weight of randomly selected product is unknown.

Weights of 16 randomly selected products in grams are given below:

195.66	202.30	205.10	200.98	189.91
198.95	201.33	193.27	192.99	196.04
197.16	207.16	197.67	202.10	204.09
193.75				

(i) Find **95%** confidence interval for mean weight of a randomly selected product and interpret the results.

(ii) Find **95%** Confidence interval for variance weight of a randomly selected product and interpret the results.

Table of Standard Normal Probabilities

Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(Z \geq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183

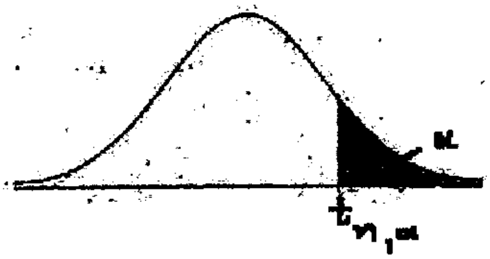

TABLE B: t-DISTRIBUTION CRITICAL VALUES

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850

Note: **When the d.f > 20, t- distribution is approximated to standard normal distribution.**

Table of $\chi^2_{\alpha,\nu}$ quantiles (χ^2 table)

df	α							
ν	0.99	0.975	0.95	0.90	0.1	0.05	0.025	0.01
10	2.558	3.247	3.94	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.92	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.66	5.629	6.571	7.79	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409
18	7.015	8.231	9.39	10.865	25.989	28.869	31.526	34.805
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191
20	8.26	9.591	10.851	12.443	28.412	31.41	34.17	37.566

Let $X \sim \chi^2_\nu$ and α be a probability. This table contains the upper α quantiles $\chi^2_{\alpha,\nu}$ of the χ^2_ν distributions such that $\Pr(X > \chi^2_{\alpha,\nu}) = \alpha$. For example, $\chi^2_{0.025,10} = 20.483$.